Longitudinal Oscillations of Multi-Story Building with Spatial Foundation Based on Continual Plate Spatial Model

Dilshodjon Shamsiyev[1, a)], Shukhrat Askarhodjaev [1, b)]
and Makhsut Kurbanbayev[1, d)]

[1] *Institute of Mechanics and Seismic Stability of Structures named after M.T. Urazbaev, Uzbekistan Academy of Sciences, Tashkent, Uzbekistan*

[a)] Corresponding author: dilshodshamsiyev.1993@mail.ru
[b)] shuhrat1608@mail.ru
[c)] kurbanbayev93@b.ru

Abstract. The article presents a numerical solution to the dynamic problem of longitudinal oscillations of a multi-story building with a spatial foundation based on a spatial continuous plate model developed using the bimoment theory of plates. The main equations of longitudinal oscillations, boundary and contact conditions of the building are given. Numerical results of displacement calculations during longitudinal oscillations of multi-story buildings for various variants of geometric dimensions are obtained.

INTRODUCTION

The increase in urban population density requires the use of optimal structural systems for multi-story civil buildings; however, despite the large number of studies on the rationality of their use, the question of choosing an assessment of the seismic resistance of structural systems for multi-story civil buildings remains open. Ensuring spatial rigidity of multi-story buildings under external loads comes down to solving complex problems associated with one of the types of dynamic calculations – modal analysis, which primarily solves issues of determining the displacement of longitudinal oscillations of structures, as a multi-story building with a spatial foundation as a whole.

The article [1] presents the results of a comparative analysis of seismic resistance of five different structural systems of multi-story civil buildings (floor height – 3 m, number of floors – 20): frame-wall, frame-barrel, barrel-wall, frame-barrel-diaphragm, frame-barrel-shell. The sum of the effective modal masses taken into account in the calculation was no less than 90% of the total mass of the system excited in the direction of seismic action for horizontal impacts, and no less than 75% for vertical impact. In article [2] is devoted to calculation method development to obtain response of structure subjected to multicomponent seismic load. Maximal and minimal «envelope» spectra were obtained to evaluate maximum and minimum structure responses respectively. Comparison of results obtained using common methods with results obtained using proposed method is presented. The average difference between the results obtained by calculating the maximum and minimum time-domain responses of the system and the results obtained using the envelope spectra was less than 5%. The article [3] considers the experience of using large-panel buildings in seismic zones. Examples of the implementation of various self-protection systems in the structures of large-panel buildings of various series are given. It is shown that the use of additional dry friction elements in the joints of panels allows to reduce seismic loads on buildings by 30% and, accordingly, the forces in the prefabricated elements of a large-panel building. The article [4] considers the organization and implementation of dynamic tests of a multi-story residential panel building in Krasnoyarsk. Based on the results of dynamic tests, the actual natural (resonant) frequencies and their vibration modes for building structures were determined. A common practice is to design high-rise buildings in such a way that the safety margin left after the formation of plastic hinges is intended to withstand earthquakes, avoiding structural collapse [5].

The studies in [6, 7] are devoted to solving problems of plate bending within the framework of Reissner's theory, in which the finite element method is used to calculate square fixed and hinged plates under uniformly distributed load.

The work [8] considers the analytical calculation of the brickwork of a barrel vault, the structure of the material of which has a pronounced variability of elastic constants. The mathematical solution of the fourth-order partial differential equation with two variables for an anisotropic orthotropic body in polar coordinates is given to create mathematical models describing the change in the elastic modulus of the arch material. Based on the solution of the problem of anisotropy of a curvilinear orthotropic body, correlations between the elastic constants in the main directions of anisotropy are obtained.

Pushover analysis is commonly used for static inelastic analysis of a structure. In this method, the lateral load is increased with the same profile to determine the inelastic capacity of the structure. At each increment of the lateral load, the plastic hinge sequence and the elastic-plastic behaviour of each element are found without considering the dynamic structure [9].

The article [10] considers the foundations of buildings and structures made of weakly viscoelastic soils, features of the theoretical justification of their deformations. The deviation of the calculated residual pore pressures from the experimental data is no more than 5% (laboratory experiment), 7% (natural experiment). The calculation method presented in the article allows predicting the deformation of the foundations of structures made of weak water-saturated soils.

The article [11] considers the contact interaction of deformable building structures or their parts. An extension of the existing formulations of the problems of frictionless contact and contact with a known friction boundary in the form of a linear complementarity problem to the formulation of frictional contact is proposed. Ultimately, a heuristic formulation of the contact problem with friction is obtained in the form of a linear complementarity problem. The problem is solved using the step-by-step Lemke algorithm in the form of a displacement method. The results of the solutions obtained for test problems and on the ANSYS software practically coincide with the results obtained by the proposed algorithm. In [12], the free vibration of axially loaded Timoshenko beams with multiple cracks with different boundary conditions, namely, hinge-hinge, fixed-fixed, fixed-hinge and fixed-free, was studied. It was found that there is good agreement between the results obtained in this study and the results available in the literature.

In references [13, 14] are devoted to the development of a theory and method for calculating thick plates. A theory and a method were developed to assess the stress-strain state of thick plates without simplifying hypotheses within the framework of three-dimensional theory of elasticity. When constructing a theory, all components of strains and stresses arising from the nonlinearity of the law of displacement distribution along the plate thickness were taken into account. The equations of motion of the plate were constructed with respect to forces, moments and bimoments. The solution method was based on exact expressions in trigonometric functions.

The studies in [15-17] are devoted to the numerical solution of the problem of transverse vibrations of a multi-story building within the framework of a continuous plate model of a solid slab under seismic influence. As a dynamic model of the building, a cantilever anisotropic plate is proposed, the theory of which was developed within the framework of the three-dimensional dynamic theory of elasticity and considers not only structural forces and moments but also bi-moments.

MATERIALS, METHODS AND OBJECT OF STUDY

To construct a continuous model of a multi-story building, it is necessary to find its reduced elasticity and density moduli. We present formulas for determining the elastic characteristics of a continuous plate model of a multi-story building from the work [13].

Let us introduce the notation for the plate elements of the building: E_1, E_2, E_3 – elastic moduli; G_{12}, G_{13}, G_{23} – shear moduli; $\nu_{12}, \nu_{13}, \nu_{23}$ – Poisson's ratios of the plate material. To determine the components of the stress tensors. forces, moments and bimoments, we introduce elastic constants $E_{11}, E_{12}, ..., E_{33}$ – elastic constants determined through Poisson's ratios and the modulus of elasticity [13]:

$$
\begin{aligned}
&E_{11} = E_1 g_{11},\ E_{22} = E_2 g_{22},\ E_{33} = E_3 g_{33} \\
&E_{12} = E_{21} = E_1 g_{12} = E_2 g_{21},\ E_{13} = E_{31} = E_1 g_{13} = E_3 g_{31},\ E_{23} = E_{32} = E_2 g_{23} = E_3 g_{32} \\
&g_{11} = \frac{1-\nu_{23}\nu_{32}}{1-\mu^2},\ \ g_{22} = \frac{1-\nu_{13}\nu_{31}}{1-\mu^2},\ \ g_{22} = \frac{1-\nu_{12}\nu_{21}}{1-\mu^2}, \\
&g_{12} = g_{21} = \frac{\nu_{12}+\nu_{13}\nu_{32}}{1-\mu^2} = \frac{\nu_{21}+\nu_{31}\nu_{23}}{1-\mu^2},\ \ g_{13} = g_{31} = \frac{\nu_{13}+\nu_{21}\nu_{32}}{1-\mu^2} = \frac{\nu_{31}+\nu_{12}\nu_{23}}{1-\mu^2}, \\
&g_{23} = g_{32} = \frac{\nu_{23}+\nu_{13}\nu_{12}}{1-\mu^2} = \frac{\nu_{32}+\nu_{31}\nu_{21}}{1-\mu^2},\ \ \mu^2 = \nu_{12}\nu_{21}+\nu_{23}\nu_{32}+\nu_{13}\nu_{31}+2\nu_{12}\nu_{23}\nu_{31}.
\end{aligned} \tag{1}
$$

Let's introduce the reduction coefficients $\xi_{11},\ \xi_{22},\ \xi_{33},\ \xi_{12},\ \xi_{13},\ \xi_{23},\ \zeta_0$ with the help of which the elasticity, shear and density moduli of the plate model of a multi-story building are given. The given elasticity and shear moduli of the building are determined by the following formulas [13]:

$$
\begin{aligned}
&E_1^{\text{given}} = \zeta_{11} E_0,\ \ E_2^{\text{given}} = \zeta_{22} E_0,\ \ E_3^{\text{given}} = \zeta_{33} E_0, \\
&G_{12}^{\text{given}} = \zeta_{12} G_0,\ \ \ G_{13}^{\text{given}} = \zeta_{13} G_0,\ G_{23}^{\text{given}} = \zeta_{23} G_0,\ \ \ \rho_{\text{given}} = \rho_0 \varsigma_0.
\end{aligned} \tag{2}
$$

and the reduced density of the building is determined by the expression

$$
\rho_{\text{reduced}} = \varsigma_0 \rho_0. \tag{3}
$$

where E_0 - modulus of elasticity of the material of the load-bearing wall of the building. Values of the reduction coefficients $\xi_{11},\ \xi_{22},\ \xi_{33},\ \xi_{12},\ \xi_{13},\ \xi_{23},\ \zeta_0$, in general, for each cell (room) are determined depending on the size and material of the room slabs of a multi-story building.

Let's write new formulas for determining the coefficients of the reduced elasticity moduli of the discrete part of the building $\xi_{11},\ \xi_{22},\ \xi_{33},\ \xi_{12},\ \xi_{13},\ \xi_{23},\ \zeta_0$, data in [13]:

$$
\begin{aligned}
&\xi_{11} = \alpha_1 \frac{S_{11}}{S_{01}},\ \ \ \xi_{22} = \alpha_2 \frac{S_{22}}{S_{02}},\ \ \ \xi_{33} = \alpha_3 \frac{S_{33}}{S_{03}},\ \xi_{12} = \alpha_4 \frac{S_{12}}{S_{01}}, \\
&\xi_{13} =_5 \frac{h_{nep}}{b_1} \lambda^*,\ \ \xi_{23} =_6 \frac{h_2}{a_1},\ \ \zeta_0 = \alpha_0 \frac{V_1}{V_0}.
\end{aligned} \tag{4}
$$

where $S_{01},\ S_{02},\ S_{03}$ – cross-sectional areas of a building in three coordinate planes of one floor of the building; $S_{11},\ S_{22},\ S_{33}$ – total areas of cross-sections of slabs in coordinate planes forming one floor of a building; λ^* – coefficient characterizing voids in the cross-section of the floor slab. V_1 – the sum of the volumes of the slabs that make up one floor of a multi-story building. – the total external volume of one multi-story building.

It should be noted that the coefficients in the formulas (3) $\alpha_0,\ \alpha_1,\ \ \alpha_2,\dots,\alpha_6$ are determined depending on the cellular structure of the building structure. When determining the reduced elasticity and shear moduli of external walls, taking into account window openings, we apply the method given in [13], in the form of approximate formulas:

$$
\begin{aligned}
&E_1^{\text{reduced}} = E_1\left(1-\frac{\eta}{\eta_0}\right),\ E_2^{\text{reduced}} = E_2\left(1-\frac{\eta}{\eta_0}\right),\ E_3^{\text{reduced}} = E_3\left(1-\frac{\eta}{\eta_0}\right), \\
&G_{12}^{\text{reduced}} = G_{12}\left(1-\frac{\eta}{\eta_0}\right),\ \ G_{13}^{\text{reduced}} = G_{13}\left(1-\frac{\eta}{\eta_0}\right)\ \ G_{23}^{\text{reduced}} = G_{23}\left(1-\frac{\eta}{\eta_0}\right).
\end{aligned} \tag{5}
$$

where $E_1, E_2, E_3, G_{12}, G_{13}, G_{23}$ – elasticity and shear moduli of external walls, η, η_0 - coefficients that depend on the size of the opening of the wall in question.

Values of the coefficients $\xi_{11},\ \xi_{22},\ \xi_{33},\ \xi_{12},\ \xi_{13},\ \xi_{23},\ \zeta_0$ for each cell (room) of the building are defined as functions of two spatial variables, $E_0,\ G_0$ – elasticity and shear moduli of the strongest load-bearing panel of the building. Formulas (1) – (4) determine the reduced elastic moduli, as an orthotropic plate model of the building.

For multi-story buildings, the following building dimensions are specified as initial data. The height and length of a multi-story building are taken to be equal to $b = nb_1$ and a, where n number of rooms across the width of one floor, b_1 – the size of one vertical transverse wall (the height of one floor of a multi-story building is considered constant); h_1 - thickness of external longitudinal vertical load-bearing walls; h_2 – thickness of internal interior transverse vertical walls; $h_{overlap}$ – floor thickness.

To represent the values of the elasticity and density moduli, their values were calculated for the following dimensions of a multi-story building: $h_1 = 0.35m,\ h_2 = 0.20m,\ h_{\text{overlap}} = 0.2m,\ a_1 = 5m,\ b_1 = 3m,\ a = 30\,m.$ The height and length of a multi-storey building are taken to be equal, respectively. $b = nb_1$ and $a = 30\,m.$ Building width H varies. Using the initial data, the values of the coefficients of reduced elastic moduli were determined, shown in Table 1 for multi-story buildings, calculated using formulas (4).

TABLE 1. Coefficients for determining the elastic moduli of a continuous building model with given initial data.

Thickness	**Coefficients and elastic characteristics of the building**						
H (м)	ξ_0	ξ_{11}	ξ_{12}	ξ_{13}	ξ_{22}	ξ_{23}	ξ_{33}
15	0,100	0,093	0,060		0,117		
18	0,089	0,083	0,050	0,067	0,107	0,04	0,09
20	0,082	0,078	0,045		0,102		

The proposed spatial model of the building takes into account all types of deformation and stress components and, as shown in work [13], is suitable for spatial calculation of seismic resistance of buildings and structures.

STATEMENT OF THE PROBLEM

The problem of longitudinal oscillations of a multi-story building is dissymmetric problem of the bimoment theory of plate structures developed in [14]. Seismic oscillations of a multi-story building within the framework of the plate model are considered in a rectangular Cartesian coordinate system. For convenience, the origin of coordinates is located in the lower left corner of the middle surface of the continuous plate model of a multi-story building. Let's direct the axes OX_1 and OX_2 by length and height, and the axis OZ – along the width of a multi-story building (plate model).

The problem of longitudinal oscillations of the bimoment theory of plate structures consists of two equations for longitudinal and tangential forces and four additionally constructed bimoment equations for nine unknown kinematic functions:

$$\bar{u}_k = \frac{u_k^{(+)} + u_k^{(-)}}{2},\quad \bar{\psi}_k = \frac{1}{2h}\int_{-h}^{h} u_k dz,\ \ \bar{\beta}_k = \frac{1}{2h^3}\int_{-h}^{h} u_k z^2 dz,\ \ (k = 1,2),$$
$$\bar{W} = \frac{u_3^{(+)} - u_3^{(-)}}{2},\ \ \bar{r} = \frac{1}{2h^2}\int_{-h}^{h} u_3 z dz,\quad \bar{\gamma} = \frac{1}{2h^4}\int_{-h}^{h} u_3 z^3 dz. \tag{6}$$

$$\bar{u}_k = \frac{u_k^{(+)} + u_k^{(-)}}{2},\quad \bar{\psi}_k = \frac{1}{2h}\int_{-h}^{h} u_k dz,\ \ \bar{\beta}_k = \frac{1}{2h^3}\int_{-h}^{h} u_k z^2 dz,\ \ (k = 1,2),$$
$$\bar{W} = \frac{u_3^{(+)} - u_3^{(-)}}{2},\ \ \bar{r} = \frac{1}{2h^2}\int_{-h}^{h} u_3 z dz,\quad \bar{\gamma} = \frac{1}{2h^4}\int_{-h}^{h} u_3 z^3 dz. \tag{7}$$

The cargo terms of the equation of motion for the first problem are introduced $\bar{q}_k, (k = 1,2), \bar{q}_3$, which are determined by the formulas:

$$\bar{q}_k = \frac{q_k^{(+)} - q_k^{(-)}}{2},\ (k = 1,2),\quad \bar{q}_3 = \frac{q_3^{(+)} + q_3^{(-)}}{2} \tag{8}$$

Efforts n_{11}, n_{12}, n_{22} stresses $\sigma_{11}, \sigma_{12}, \sigma_{22}$ are defined by expressions:

$$n_{11} = E_{11}\bar{\varepsilon}_{11} + E_{12}\bar{\varepsilon}_{22} + E_{13}\bar{\varepsilon}_{33},$$
$$n_{22} = E_{12}\bar{\varepsilon}_{11} + E_{22}\bar{\varepsilon}_{22} + E_{23}\bar{\varepsilon}_{33}, \quad (9)$$
$$n_{12} = n_{21} = G_{12}\left(\bar{\varepsilon}_{12} + \bar{\varepsilon}_{21}\right)$$

where $\bar{\varepsilon}_{11} = \frac{\partial \bar{\psi}_1}{\partial x_1}$, $\bar{\varepsilon}_{22} = \frac{\partial \bar{\psi}_2}{\partial x_2}$, $\bar{\varepsilon}_{33} = \frac{2\bar{W}}{H}$, $\bar{\varepsilon}_{12} = \frac{\partial \bar{\psi}_2}{\partial x_1}$.

Bimoments $\bar{p}_{11}, \bar{p}_{22}, \bar{p}_{12}$ from stress $\sigma_{11}, \sigma_{12}, \sigma_{22}$ are defined as:

$$\bar{p}_{11} = E_{11}\bar{e}_{11} + E_{12}\bar{e}_{22} + E_{13}\bar{e}_{33},$$
$$\bar{p}_{12} = \bar{p}_{21} = G_{12}\bar{e}_{12}, \quad (10)$$
$$\bar{p}_{22} = E_{12}\bar{e}_{11} + E_{22}\bar{e}_{22} + E_{23}\bar{e}_{33}.$$

where $\bar{e}_{11} = \frac{\partial \bar{\beta}_1}{\partial x_1}$, $\bar{e}_{22} = \frac{\partial \bar{\beta}_2}{\partial x_2}$, $\bar{e}_{33} = \frac{2\bar{W} - 4\bar{r}}{H}$, $\bar{e}_{12} = \frac{\partial \bar{\beta}_2}{\partial x_1} + \frac{\partial \bar{\beta}_1}{\partial x_2}$.

Intensities of transverse bimoments $\bar{p}_{13}, \bar{p}_{23}$ и $\bar{\tau}_{13}, \bar{\tau}_{23}$ from shear stresses σ_{13}, σ_{23} are constructed in the form of the following expressions:

$$\bar{p}_{k3} = G_{k3}\,\bar{\varepsilon}_{k3}, \quad \bar{\tau}_{k3} = G_{k3}\,\bar{e}_{k3}, \quad (k = 1,2). \quad (11)$$

where $\bar{\varepsilon}_{k3} = \frac{\partial \bar{r}}{\partial x_k} + \frac{2(\bar{u}_k - \bar{\psi}_k)}{H}$, $\bar{e}_{k3} = \frac{\partial \bar{\gamma}}{\partial x_k} + \frac{2(\bar{u}_k - 3\bar{\beta}_k)}{H}$, $(k = 1,2)$.

Intensities of bimoments $\bar{p}_{33}$ и $\bar{\tau}_{33}$ from normal voltage σ_{33} received in the form:

$$\bar{p}_{33} = E_{31}\bar{\varepsilon}_{11} + E_{32}\bar{\varepsilon}_{22} + E_{33}\bar{\varepsilon}_{33}, \quad \bar{\tau}_{33} = E_{31}\bar{e}_{11} + E_{32}\bar{e}_{22} + E_{33}\bar{e}_{33} \quad (12)$$

The equations of motion of the plate relative to the longitudinal and tangential forces are constructed in the form:

$$\frac{\partial n_{11}}{\partial x_1} + \frac{\partial n_{12}}{\partial x_2} + \frac{2\bar{q}_1}{H} = \rho\ddot{\bar{\psi}}_1, \quad \frac{\partial n_{21}}{\partial x_1} + \frac{\partial n_{22}}{\partial x_2} + \frac{2\bar{q}_2}{H} = \rho\ddot{\bar{\psi}}_2. \quad (13)$$

Note that in the system of two equations (12)contains three unknown functions $\bar{\psi}_1$, $\bar{\psi}_2$, $\bar{W}$.

With respect to longitudinal and tangential bimoments, two equations of motion are also constructed in the form:

$$\frac{\partial \bar{p}_{11}}{\partial x_1} + \frac{\partial \bar{p}_{12}}{\partial x_2} - \frac{4\bar{p}_{13}}{H} + \frac{2\bar{q}_1}{H} = \rho H\ddot{\bar{\beta}}_1$$
$$\frac{\partial \bar{p}_{12}}{\partial x_1} + \frac{\partial \bar{p}_{22}}{\partial x_2} - \frac{4\bar{p}_{23}}{H} + \frac{2\bar{q}_2}{H} = \rho\ddot{\bar{\beta}}_2. \quad (14)$$

In contrast to traditional plate theories, two more equations of plate motion are constructed relative to the intensity of transverse bimoments in the following form:

$$\frac{\partial \bar{p}_{13}}{\partial x_1} + \frac{\partial \bar{p}_{23}}{\partial x_2} - \frac{2\bar{p}_{33}}{H} + \frac{2\bar{q}_3}{H} = \rho\ddot{\bar{r}} \quad (15)$$

$$\frac{\partial \bar{\tau}_{13}}{\partial x_1} + \frac{\partial \bar{\tau}_{23}}{\partial x_2} - \frac{6\bar{\tau}_{33}}{H} + \frac{2\bar{q}_3}{H} = \rho\ddot{\bar{\gamma}} \quad (16)$$

Using the method of expansion of displacements into an infinite Malaren series, three more equations of longitudinal oscillations of a plate model of a multi-story building with respect to generalized displacement functions were constructed in [14] $\bar{u}_1$, $\bar{u}_2, \bar{W}$ points of the external walls in the following form:

$$
\begin{aligned}
&(\tilde{u}_1)_{N,M} = 21(\tilde{\beta}_1)_{N,M} - 7(\tilde{\psi}_1)_{N,M}, \quad (\tilde{u}_2)_{N,M} = 21(\tilde{\beta}_2)_{N,M} - 7(\tilde{\psi}_2)_{N,M}, \\
&(\tilde{W})_{N,M} = \frac{1}{2}(21\tilde{\gamma} - 7\tilde{r})_{N,M}.
\end{aligned} \tag{17}
$$

$$
(\tilde{W})_{N,M} = \frac{1}{2}(21\tilde{\gamma} - 7\tilde{r}) - \frac{1}{30}H\left[\frac{E_{31}}{E_{33}}\frac{\partial \bar{u}_1}{\partial x_1} + \frac{E_{32}}{E_{33}}\frac{\partial \bar{u}_2}{\partial x_2} - \frac{E_{61}}{E_{33}}\left(\frac{\partial \tilde{u}_1}{\partial x_1} + \frac{\partial \tilde{u}_2}{\partial x_2}\right)\right] + \frac{H\bar{q}}{30E_{33}} \tag{18}
$$

When describing the boundary conditions for the equations of longitudinal oscillations of buildings (6)-(17), in addition to forces, moments and bimoments, we also introduce specific bimoments $\bar{\sigma}_{11}, \bar{\sigma}_{12}, \bar{\sigma}_{22}, \bar{\sigma}_{11}^*, \bar{\sigma}_{22}^*$, which are determined by the formulas obtained in [14]. Bimoments $\bar{\sigma}_{11}, \bar{\sigma}_{12}, \bar{\sigma}_{22}$ are determined by formulas:

$$
\begin{aligned}
&\bar{\sigma}_{11} = \left(E_{11} - \frac{E_{13}}{E_{33}}E_{31}\right)\frac{\partial \bar{u}_1}{\partial x_1} + \left(E_{12} - \frac{E_{13}}{E_{33}}E_{32}\right)\frac{\partial \bar{u}_2}{\partial x_2} + \frac{E_{13}}{E_{33}}\bar{q}_3, \\
&\bar{\sigma}_{22} = \left(E_{21} - \frac{E_{23}}{E_{33}}E_{31}\right)\frac{\partial \bar{u}_1}{\partial x_1} + \left(E_{22} - \frac{E_{23}}{E_{33}}E_{32}\right)\frac{\partial \bar{u}_2}{\partial x_2} + \frac{E_{23}}{E_{33}}\bar{q}_3, \\
&\bar{\sigma}_{12} = G_{12}\left(\frac{\partial \bar{u}_1}{\partial x_2} + \frac{\partial \bar{u}_2}{\partial x_1}\right).
\end{aligned} \tag{19}
$$

Let us write down the expressions for the intensities of the bimoments $\bar{\sigma}_{11}^*, \bar{\sigma}_{22}^*$ to describe longitudinal oscillations, determined by the formulas:

$$
\begin{aligned}
&\bar{\sigma}_{11}^* = -E_{11}\frac{\partial^2 \bar{W}}{\partial x_1^2} - E_{12}\frac{\partial^2 \bar{W}}{\partial x_1^2} + E_{13}\frac{420(\bar{W} + 6\bar{r} - 15\bar{\gamma})}{H}, \\
&\bar{\sigma}_{22}^* = -E_{12}\frac{\partial^2 \bar{W}}{\partial x_1^2} - E_{22}\frac{\partial^2 \bar{W}}{\partial x_1^2} + E_{23}\frac{420(\bar{W} + 6\bar{r} - 15\bar{\gamma})}{H},
\end{aligned} \tag{20}
$$

Next, we define the boundary conditions for the problem under consideration on longitudinal oscillations of multi-story buildings.

On the free lateral faces of the building we have the conditions of equality to zero of forces, moments and bimoments and force factors:

$$
\begin{aligned}
&n_{11} = 0, \ n_{12} = 0, \ \bar{p}_{11} = 0, \ \bar{p}_{12} = 0, \ \bar{p}_{13} = 0, \\
&\bar{\tau}_{13} = 0, \ \bar{\sigma}_{11} = 0, \ \bar{\sigma}_{12} = 0 \ \bar{\sigma}_{13}^* = 0.
\end{aligned} \tag{21, a}
$$

On the free upper edge of the building we have the following conditions:

$$
\begin{aligned}
&n_{12} = 0, \ n_{22} = 0, \ \bar{p}_{12} = 0, \ \bar{p}_{22} = 0, \ \bar{p}_{23} = 0, \\
&\bar{\tau}_{23} = 0, \ \bar{\sigma}_{12} = 0, \ \bar{\sigma}_{22} = 0 \ \bar{\sigma}_{23}^* = 0.
\end{aligned} \tag{21, b}
$$

On the side faces of the building located in the ground we have conditions of equality of force factors and loads from the ground:

$$
\begin{aligned}
&n_{11} = n_{11}^{\text{гр}}, \ n_{12} = n_{12}^{\text{гр}}, \ \bar{p}_{11} = p_{11}^{\text{гр}}, \ \bar{p}_{12} = p_{12}^{\text{гр}}, \ \bar{p}_{13} = p_{13}^{\text{гр}}, \ \bar{\tau}_{13} = \tau_{12}^{\text{гр}}, \\
&\bar{\sigma}_{11} = \sigma_{12}^{\text{гр}}; \ \ \bar{\sigma}_{12} = \sigma_{12}^{\text{гр}} \ \bar{\sigma}_{13}^* = \sigma_{12}^{\text{гр}}.
\end{aligned} \tag{22, a}
$$

On the free upper edge of the building we have the conditions:

$$
\begin{aligned}
&n_{12} = n_{12}^{\text{гр}}, \ n_{22} = n_{22}^{\text{гр}}, \ \bar{p}_{12} = p_{11}^{\text{гр}}, \ \bar{p}_{22} = p_{22}^{\text{гр}}, \ \bar{p}_{23} = p_{23}^{\text{гр}}, \ \bar{\tau}_{23} = \tau_{23}^{\text{гр}}, \\
&\bar{\sigma}_{12} = \sigma_{12}^{\text{гр}}; \ \ \bar{\sigma}_{22} = \sigma_{22}^{\text{гр}} \ \bar{\sigma}_{23}^* = \sigma_{23}^{\text{гр}}.
\end{aligned} \tag{22, b}
$$

When considering longitudinal oscillations of a building, the maximum stresses between longitudinal and transverse walls are determined by the formulas:

$$\bar{\sigma}_{13}^{*} = G_{13}\left[\frac{5}{6}\frac{\partial \overline{W}}{\partial x_1} + \frac{E_{31}}{E_{33}}\frac{H}{36}\frac{\partial^2 \bar{u}_1}{\partial x_1^2} + \frac{H}{36}\frac{E_{32}}{E_{33}}\frac{\partial^2 \bar{u}_2}{\partial x_1 \partial x_2} - \frac{35\left(9\bar{\beta}_1 - 2\bar{\psi}_1 - \bar{u}_1\right)}{6H}\right],$$
$$\bar{\sigma}_{13}^{*} = G_{23}\left[\frac{5}{6}\frac{\partial \overline{W}}{\partial x_2} + \frac{E_{31}}{E_{33}}\frac{H}{36}\frac{\partial^2 \bar{u}_1}{\partial x_2 \partial x_1} + \frac{H}{36}\frac{E_{32}}{E_{33}}\frac{\partial^2 \bar{u}_2}{\partial x_2^2} - \frac{35\left(9\bar{\beta}_2 - 2\bar{\psi}_2 - \bar{u}_2\right)}{6H}\right], \quad (23)$$

$$\bar{\sigma}_{33}^{*} = \frac{7E_{31}}{6}\frac{\partial \bar{u}_1}{\partial x_1} + \frac{7E_{32}}{6}\frac{\partial \bar{u}_2}{\partial x_2} - \frac{H}{36}\left(E_{31}\frac{\partial^2 \overline{W}}{\partial x_1^2} + E_{32}\frac{\partial^2 \overline{W}}{\partial x_2^2}\right) - E_{33}\frac{35\left(33\bar{\gamma} - 9\overline{W} - 4\bar{r}\right)}{6H}. \quad (24)$$

The methodology and algorithm for the numerical solution of the problem of oscillations of a multi-story building with longitudinal oscillations are developed based on the finite difference method. To approximate the derivatives of displacements by spatial coordinates, we will use the formulas of central difference schemes. When approximating the derivatives of stresses, forces, moments and bimoments, central finite difference schemes on half steps are used (Figure 1), which have the second order of accuracy:

$$\frac{\partial F_{i,j}^{k}}{\partial x_1} = \frac{F_{i+\frac{1}{2},j}^{k} - F_{i-\frac{1}{2},j}^{k}}{\Delta x_1}, \quad \frac{\partial F_{i,j}^{k}}{\partial x_2} = \frac{F_{i,j+\frac{1}{2}}^{k} - F_{i,j-\frac{1}{2}}^{k}}{\Delta x_2} \quad \left(i = 1, N;\ j = 1, M\right) \quad (25)$$

In here $\Delta x_1 = \frac{a}{N}$, $\Delta x_2 = \frac{b}{M}$ – grid method calculation step, N, M – number of divisions per grid.

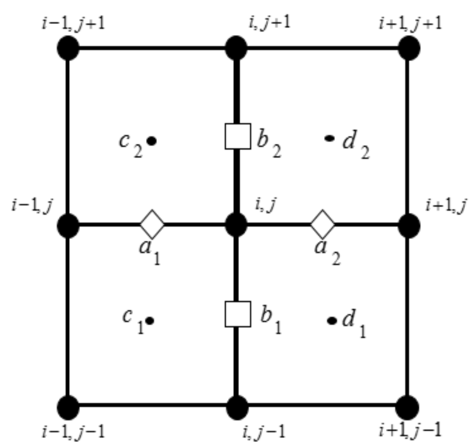


FIGURE 1. Finite difference approximation of the derivatives of force factors and displacements.

To approximate the derivatives of stresses, forces, moments and bimoments, central finite-difference schemes on half-steps are used, which have the second order of accuracy. The conditions for the equality to zero of the force factors of a multi-story building at free edges are approximated as the equality to zero of the arithmetic mean value of the displacements of external and internal points.

The program for calculating the displacements and force factors of a multi-story building is compiled in the Delphi algorithmic environment.

ANALYSIS OF NUMERICAL RESULTS

Numerical calculations are made under the assumption that seismic soil movement occurs in the direction of the *OZ* axis (along the width of the building) in the form of acceleration of the building base:

$$\ddot{u}_0(t) = a_0 \cos(\omega_0 t) \quad (26)$$

where $a_0 = k_c g$ - maximum acceleration and $\omega_0 = 2\pi\nu_0$ – circular frequency of the soil base, k_c и ν_0 - the earthquake intensity coefficient and the natural frequency of the external impact, respectively.

From here we obtain the displacement of the building foundation in the forme:

$$u_0(t) = \frac{A_0}{2}(1 - \cos(\omega_0 t)). \qquad (27)$$

In here $A_0 = \frac{2k_c g}{\omega_0^2}$. – amplitude of the base displacement. We take zero values as the initial conditions. Note that the seismicity coefficients for seven-point, eight-point and nine-point earthquakes are equal to $k_c = 0.1,\ 0.2,\ 0.4$, respectively.

We assume that the external walls are made of reinforced concrete with a modulus of elasticity $E_0 = 30000$ MPa density $\rho_0 = 2500$ kg/m^3, Poisson's ratio $\nu_0 = 0.3$. Внутренние We consider the walls to be made of expanded clay concrete with the following physical characteristics: modulus of elasticity $E = 7500$ MPa, density $\rho = 1200$ kg/m^3, Poisson's ratio $\nu = 0.3$. The foundation of the building consists of reinforced concrete with a modulus of elasticity $E_{Found} = 2500$ MPa, density $\rho_{Found} = 2500$ kg/m^3, Poisson's ratio $\nu_{Found} = 0.3$. Foundation width $h_{Found} = 1.2$ m.

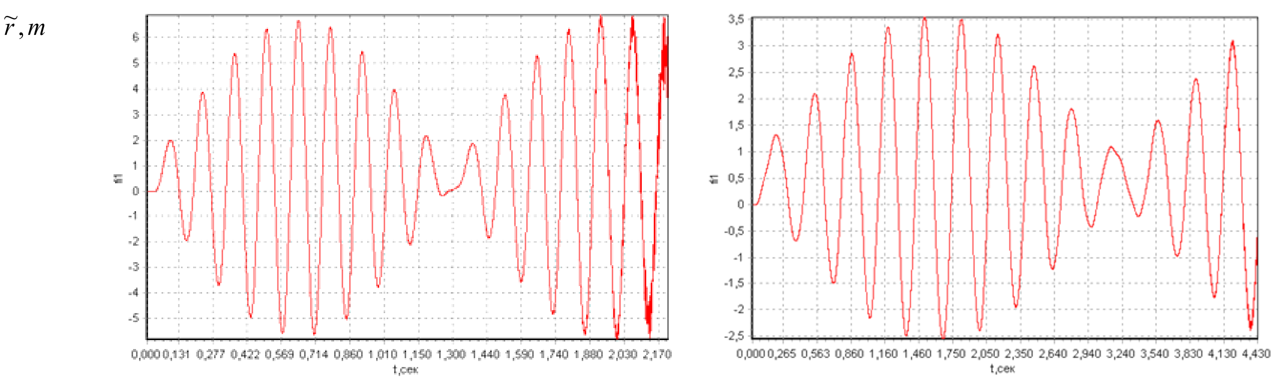
The results of calculations of forced oscillations of a building within the framework of a plate structure model are presented for the following dimensions of the building slabs:

$$h_1 = 0.40m,\ h_2 = 0.20m,\ h_{\text{overlap}} = 0.2m,\ a_1 = 5m,\ b_1 = 3m.$$

The height and length of a multi-storey building are taken to be equal, respectively. $b = nb_1$ and $a = 30\,м$, and the width of the building H varies. The height for the ninth, twelfth and sixteenth floors of a sixteen-story building is taken to be equal to, respectively, *b=30 m, b =40 m* and *b =51 m.*

Using the initial data, the values of the reduced elasticity, shear and density moduli presented in Table 1 for a sixteen-story building. We present the numerical results obtained using the developed methodology and algorithm for calculating multi-story buildings under longitudinal seismic impacts. Note that the maximum stress values are determined in the external walls of a large-panel multi-story building. The amplitude of the external impact A_0 depends on the magnitude of the earthquake, which is determined from the condition $A_0\omega_0^2 = k_c g$, where k_c, g - seismicity coefficient and acceleration of gravity. Then the amplitude of the external impact will be equal to $A_0 = k_c g / \omega_0^2$.

Figure 2 shows the graphs of changes in normal displacement. φ_1 and transverse compression functions $\bar{r}$ by time in the middle of the sixteenth floor of a sixteen-story building during a 7-point earthquake.



a) normal displacement change graph φ_1 *b) transverse compression functions* $\bar{r}$

FIGURE 2. Graphs of changes in normal displacement and transverse compression functions by time in the middle of the sixteenth floor of a sixteen-story building.

From the graph (Fig. 2, a) it is clear that in the middle of the sixteenth floor of the building the maximum displacement equal. The maximum value of compression of the midpoint of the sixteenth floor was equal to (Fig. 2, b).

Figure 3 shows graphs of changes in time t of the values of normal displacement at the highest point of a sixteen-story building near resonant oscillations with a frequency p1=3.001 Hz.

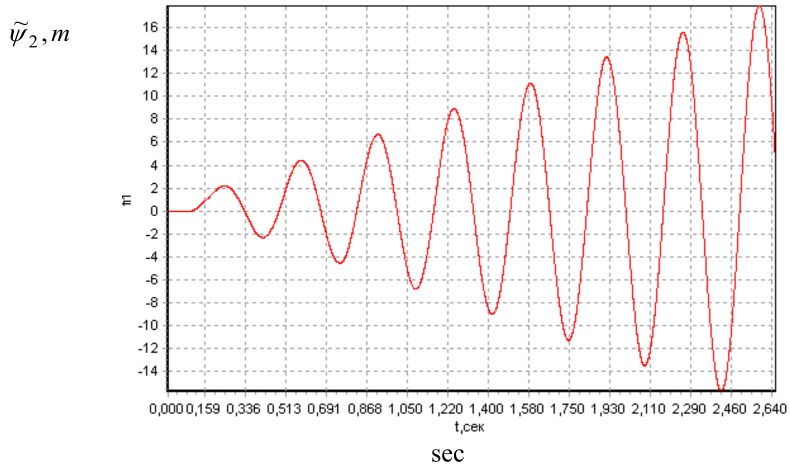


FIGURE 3. Normal displacement graph at the midpoint of the upper level of a sixteen-story building near resonant oscillations with a frequency of p1=3.001 Hz.

Table 2 shows the values of longitudinal displacement at the midpoints of various floors of a sixteen-story building.

TABLE 2 Minimum and maximum displacement values φ_1 sixteen-story building with external influence frequency $\nu_0 = 3.3\ Hz$ at different points on the floors

b, m	Number of floors in a sixteen-story building.	φ_1.	
		min	max
27	9	-2,502	3,501
40	12	-5,051	6,050
51	16	-7,100	8,111

Displacement distribution fields, arising under the action of seismic load are obtained dissymmetrically. The analysis of numerical results showed that with an increase in the height of the floor, horizontal displacements increase significantly. It should be noted that the functions of transverse compression significantly less than the transverse shear values.

CONCLUSION

According to formulas, (1) - (4) the reduced elastic moduli are less than the elastic modulus of the panels by approximately 8–15 times, and the reduced density of the plate model of the discrete part of the building is 7–12 times less than the density of the panel material. Such a discrepancy in modules is explained by the presence of a large number of voids and the cellular structure of the building. Thus, a continuum model, a method and an algorithm for numerical solution of the seismic resistance problem, as well as methods for determining displacements during longitudinal oscillations of a multi-story building are proposed. With an increase in the height of the floor, horizontal displacements increase significantly.

ACKNOWLEDGMENTS

The authors express their gratitude to the Institute of Mechanics and Seismic Stability of Structures named after M.T.Urazbaev, Uzbekistan Academy of Sciences. for the financial assistance provided in conducting experiments, developing a program for calculating transverse oscillations of multi-story buildings and preparing the article.

CONFLICTS OF INTEREST

This research work was carried out within the framework of research programs implemented in 2025 at the Institute of Mechanics and Seismic Stability of Structures named after M.T.Urazbaev, Uzbekistan Academy of Sciences, financed from the state budget. All authors declare that there are no competing interests.

REFERENCES

1. V. Shirokov, I. Kholopov and A. Solovejv, Journal of Procedia Engineering **153**, 655-661 (2016).
2. T.N.T. Chik, M.F. Zakaria, M.A. Remali and N.A. Yusof, "Vibration response of multi storey building using finite element modelling," in *Soft Soil Engineering International Conference-2015*, IOP Conference 136, edited by D.C.Wijeyesekera *et al*. (IOP Publishing, Batu Pahat, 2016), 012037.
3. V.O. Mkrtycheva, G.A. Dzhinchvelashvilia and M.S. Busalova, Journal of Procedial Engineering **111,** 545–549 (2015).
4. S. Ramhormozian, G. Clifton, T. Good, S. Jiang and G.A. MacRae, "Dynamic time history analysis of a low damage multi-storey building incorporating the seismic friction dampers using a proposed simplified multi degree of freedom model: Is selfcentring really a concern," in *Pacific Conference on Earthquake Engineering and Annual Conference-2019*, Pacific Conference 253, edited by A. John *et al*. (NZSEE publishing, New Zealand, 2019), 253.
5. J. Goncharova, Z. Abobakirova, S. Davlyatov, S. Umarov and A. Mukhamedzanov, "Polymer reagent in construction practice," in *IV International Scientific Conference Construction Mechanics, Hydraulics and Water Resources Engineering-2022*, E3S Web of Conferences 365, edited by R. Bennacer (EDP Sciences, Tashkent, 2023), 02024.
6. Y.Y. Tyukalov, Journaol of Civil Engineering **67**, 39-53 (2016).
7. Y.Y. Tyukalov, Journaol of Civil Engineering **89**, 61-68 (2019).
8. N. Ganesh, B. Sushma, C. Lokeswar Reddy, S. Rafiulla Khadri and M. Thaheer Basha, *Dynamic Analysis of Multi-Storey Building* (Civil Engineering, Nandyal, 2023), pp. 20–45.
9. S. Kim, Y. Ju and W. Hong, *Seismic Evaluation of High-Rise Buildings by Modified Dynamic Inelastic Analysis Method* (Department of Architectural Engineering, Seoul, 2000) pp. 8-15.
10. Y.K. Ju, W.K. Hong, S.D. Kim and C.L. Park, Journal of KSSC **10**, 115-123 (1998).
11. D.D. Angela, G. Magliulo and E. Cosenza, Structural Safety **93**, 102126 (2021).
12. J.M. Bracci, S.K. Kunnath, and A.M. Reinhorn, Journal of ASCE **123**, 3-10 (1997).
13. M. Usarov, A. Salokhiddinov, D. Usarov, I. Khazratkulov and N. Dremova, "To the theory of bending and oscillations of three-layered plates with a compressible filler" in *International Conference Reliability of buildings and constructions - 2020*, IOP Conference 896, edited by D. Nazarov (IOP Publishing, Tashkent, 2020), 052037.
14. M. Mirsaidov, M. Usarov, "Bimoment theory construction to assess the stress state of thick orthotropic plates" in *International Scientific Conference "Earth and Environmental Science" – 2020,* E3S Web of Conferences, 971, edited D. Nazarov, (EDP Sciences, Tashkent, 2020), 032041.
15. M. Usarov, D. Usarov and G. Mamatisaev, "Calculation of a Spatial Model of a Box-Type Structure in the LIRA Design System Using the Finite Difference Method," in *XV International Online Conference Improving Farming Productivity And Agroecology–Ecosystem Restoration–2023,* E3S Web of Conferences 403, edited by R. Bennacer (EDP Sciences, Tashkent, 2023), pp.1267–1275.
16. M. Usarov, G. Mamatisaev, J. Yarashov, and E. Toshmatov, "Non-stationary oscillations of a box-like structure of a building" in in *International Scientific Conference Modelling and Methods of Structural Analysis – 2019,* IOP Conference 1425, edited by A. Volkov *et al*. (IOP Publishing, Moscow, 2019), 012003.
17. K.D. Salyamova, A.S. Yuvmitov, D.K. Shamsiev, G.T. Ayubov, A. Numonov and I. Khazratkulov, "Seismic resistance evaluation of multi-story buildings using the modern LIRA-SAPR SP" in *International Scientific Siberian Transport Forum-2023*, E3S Web of Conferences 402, edited by R. Bennacer (EDP Sciences, Tashkent, 2023), 07021.