Transverse Oscillations of a Multi-Story Building with Spatial Foundations Based on a Continual Plate Spatial Model

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Abstract. The article presents a numerical solution to the dynamic problem of transverse oscillations of a multi-story building with spatial foundations within the framework of a spatial continuous plate model developed using the bimoment theory of thick plates. The main equations of transverse oscillations, boundary conditions on the lateral and upper faces of the building are given. Numerical results of calculations of displacements and stresses during transverse oscillations of multi-story buildings for various variants of geometric dimensions are obtained.

INTRODUCTION

Calculation of buildings and structures with spatial foundations for strength is one of the current complex and important tasks. Today, many scientists and researchers are engaged in improving existing methods for calculating high-rise buildings for seismic strength, as well as developing new spatial methods.

The article [1] proposes a new approximate method based on analytical formulas for estimating the ultimate strength of reinforced panels by studying the mechanisms of panel failure. The results of calculations performed by the proposed method and the finite element method are compared. Very good agreement is obtained for all failure scenarios studied.

Reference [2] investigated the natural frequency and modes of vibration of a multi-story office building with a foundation system and analysed the influence of the vibration mode on the dynamic characteristics of the structure.

In [3], the calculation of a multi-storey monolithic concrete building for an earthquake is discussed. The problem is solved in the time domain using the direct dynamic method. The study in [4] presents the results of a dynamic analysis of an 11-story steel moment-resisting building with sliding pin connections as beam-column connections, considering influential self-cantering factors such as the continuity of MRFs and gravity columns and column base, and the flexibility of the diaphragm.

In [5], the dynamic characteristics and oscillations of various axisymmetric and flat structures are considered, taking into account different geometries, spatial factors and inelastic properties of materials.

In [6, 7], the analytical calculation of the brickwork of a barrel vault, the material structure of which has a pronounced variability of elastic constants, is considered. A mathematical solution is given to a fourth-order partial differential equation with two variables for an anisotropic orthotropic body in polar coordinates to create mathematical models describing the change in the elastic modulus of the vault material.

In [8], the foundations of buildings and structures made of weakly viscoelastic soils and the features of the theoretical justification of their deformations are considered. The need for this study is due to the discrepancy between the theory of filtration compaction and field and laboratory experiments.

In [9], the free vibration of axially loaded Timoshenko beams with multiple cracks with different boundary conditions, namely, hinged-hinge, fixed-fixed, fixed-hinge and fixed-free, is studied. This article studies in detail the influence of crack depth, number of cracks, crack position, axial load, shear strain and rotational inertia on the dynamic behavior of beams with multiple cracks.

The steps to perform a modified dynamic inelastic analysis are described, and the proposed method is applied to examples of seven- and fifteen-story buildings. A common practice is to design high-rise buildings in such a way that the safety margin available after the formation of plastic hinges is designed to withstand earthquakes, avoiding the collapse of structures [10].

Article [11] presents an advanced seismic damage assessment of unsupported non-structural components through rigid block analysis. At each increment of lateral load, the plastic hinge sequence and the elastoplastic behaviour of each element were determined without dynamic consideration of the structure [12].

In references [13, 14] are devoted to the development of a theory and method for calculating thick plates. A theory and a method were developed to assess the stress-strain state of thick plates without simplifying hypotheses within the framework of three-dimensional theory of elasticity. When constructing a theory, all components of strains and stresses arising from the nonlinearity of the law of displacement distribution along the plate thickness were taken into account. The equations of motion of the plate were constructed with respect to forces, moments and bimoments. The solution method was based on exact expressions in trigonometric functions.

The studies in [15-17] are devoted to the numerical solution of the problem of transverse vibrations of a multi-story building within the framework of a continuous plate model of a solid slab under seismic influence. As a dynamic model of the building, a cantilever anisotropic plate is proposed, the theory of which was developed within the framework of the three-dimensional dynamic theory of elasticity and considers not only structural forces and moments but also bi-moments.

MATERIALS, METHODS AND OBJECT OF STUDY

To construct a continuous model of a multi-story building, it is necessary to find its reduced elasticity and density moduli. We present formulas for determining the elastic characteristics of a continuous plate model of a multi-story building from the work [14].

Let us introduce the notation for the plate elements of the building: $E_1, E_2,\ E_3$ – elastic moduli; G_{12}, G_{13}, G_{23} – shear moduli; $\nu_{12}, \nu_{13}, \nu_{23}$ – Poisson's ratios of the plate material. To determine the components of the stress tensors. forces, moments and bimoments, we introduce elastic constants $E_{11},\ E_{12},..., E_{33}$ – elastic constants determined through Poisson's ratios and the modulus of elasticity [14]:

$$
\begin{aligned}
&E_{11} = E_1 g_{11},\ E_{22} = E_2 g_{22},\ E_{33} = E_3 g_{33},\\
&E_{12} = E_{21} = E_1 g_{12} = E_2 g_{21},\ E_{13} = E_{31} = E_1 g_{13} = E_3 g_{31},\ E_{23} = E_{32} = E_2 g_{23} = E_3 g_{32},\\
&g_{11} = \frac{1-\nu_{23}\nu_{32}}{1-\mu^2},\ \ g_{22} = \frac{1-\nu_{13}\nu_{31}}{1-\mu^2},\ \ g_{22} = \frac{1-\nu_{12}\nu_{21}}{1-\mu^2},\\
&g_{12} = g_{21} = \frac{\nu_{12}+\nu_{13}\nu_{32}}{1-\mu^2} = \frac{\nu_{21}+\nu_{31}\nu_{23}}{1-\mu^2},\ g_{13} = g_{31} = \frac{\nu_{13}+\nu_{21}\nu_{32}}{1-\mu^2} = \frac{\nu_{31}+\nu_{12}\nu_{23}}{1-\mu^2},\\
&g_{23} = g_{32} = \frac{\nu_{23}+\nu_{13}\nu_{12}}{1-\mu^2} = \frac{\nu_{32}+\nu_{31}\nu_{21}}{1-\mu^2}, \quad \mu^2 = \nu_{12}\nu_{21} + \nu_{23}\nu_{32} + \nu_{13}\nu_{31} + 2\nu_{12}\nu_{23}\nu_{31}
\end{aligned}
\tag{1}
$$

Let's introduce the reduction coefficients $\xi_{11},\ \xi_{22},\ \xi_{33},\ \xi_{12},\ \xi_{13},\ \xi_{23},\ \zeta_0$, with the help of which the elasticity, shear and density moduli of the plate model of a multi-story building are given. The given elasticity and shear moduli of the building are determined by the following formulas [14]:

$$
\begin{aligned}
&E_1^{\text{reduced}} = \zeta_{11} E_0,\ \ E_2^{\text{reduced}} = \zeta_{22} E_0,\ \ E_3^{\text{reduced}} = \zeta_{33} E_0,\\
&G_{12}^{\text{reduced}} = \zeta_{12} G_0,\ \ G_{13}^{\text{reduced}} = \zeta_{13} G_0,\ G_{23}^{\text{reduced}} = \zeta_{23} G_0.
\end{aligned}
\tag{2}
$$

and the reduced density of the building is determined by the expression:

$$\rho_{\text{reduced}} = \varsigma_0 \rho_0. \tag{3}$$

where E_0 - modulus of elasticity of the material of the load-bearing wall of the building.

Values of the reduction coefficients $\xi_{11},\ \xi_{22},\ \xi_{33},\ \xi_{12},\ \xi_{13},\ \xi_{23},\ \zeta_0$, in general, for each cell (room) are determined depending on the size and material of the room slabs of a multi-story building.

Let's write new formulas for determining the coefficients of the reduced elasticity moduli of the discrete part of the building $\xi_{11}, \xi_{22}, \xi_{33}, \xi_{12}, \xi_{13}, \xi_{23}, \zeta_0$, data in [14]:

$$\xi_{11} = \alpha_1 \frac{S_{11}}{S_{01}}, \quad \xi_{22} = \alpha_2 \frac{S_{22}}{S_{02}}, \quad \xi_{33} = \alpha_3 \frac{S_{33}}{S_{03}}, \xi_{12} = \alpha_4 \frac{S_{12}}{S_{01}},$$
$$\xi_{13} = \alpha_5 \frac{h_{\text{overlap}}}{b_1} \lambda^*, \quad \xi_{23} = \alpha_6 \frac{h_2}{a_1}, \quad \zeta_0 = \alpha_0 \frac{V_1}{V_0}. \tag{4}$$

Here S_{01}, S_{02}, S_{03}–cross-sectional area of the building in three coordinate planes of one floor of the building; S_{11}, S_{22}, S_{33}–total cross-sectional areas of slabs in coordinate planes forming one floor of the building; λ^*– coefficient characterizing the voids in the cross section of the floor slab. Coefficient α is determined depending on the cell-type structure of the building structure. V_1 – the sum of the volumes of the slabs that make up one floor of a multi-story building. V_0 – the total external volume of one multi-story building.

Depending on the size of the slabs, rooms, and the building itself, the above areas are determined using the methodology presented in [14], in the following form:

$$S_{01} = E_0 b_1 H, \quad S_{02} = E_0 a H, \quad S_{03} = E_0 a b_1, \tag{5}$$

here G_{fshm}–building floor shear modulus; G_2–internal wall shear modulus; $E_b^{(2)}$–modulus of elasticity of internal walls; E_{fem}–floor elastic modulus.

It should be noted that the coefficients in the formulas (4) $\alpha_0, \alpha_1, \alpha_2,..., \alpha_6$ are determined depending on the cellular structure of the building structure. When determining the reduced elasticity and shear moduli of external walls, taking into account window openings, we apply the method given in [14], in the form of approximate formulas:

$$E_1^{reduced} = E_1\left(1 - \frac{\eta}{\eta_0}\right), \; E_2^{reduced} = E_2\left(1 - \frac{\eta}{\eta_0}\right), \; E_3^{reduced} = E_3\left(1 - \frac{\eta}{\eta_0}\right),$$
$$G_{12}^{reduced} = G_{12}\left(1 - \frac{\eta}{\eta_0}\right), \; G_{13}^{reduced} = G_{13}\left(1 - \frac{\eta}{\eta_0}\right) \; G_{23}^{reduced} = G_{23}\left(1 - \frac{\eta}{\eta_0}\right). \tag{6}$$

where $E_1, E_2, E_3, G_{12}, G_{13}, G_{23}$– elasticity and shear moduli of external walls, η, η_0 - constant coefficients that depend on the size of the opening of the wall in question.

Values of the coefficients $\xi_{11}, \xi_{22}, \xi_{33}, \xi_{12}, \xi_{13}, \xi_{23}, \zeta_0$ for each cell (room) of the building are defined as functions of two spatial variables, E_0, G_0 – elasticity and shear moduli of the strongest load-bearing panel of the building. Formulas (1) – (6) determine the reduced elastic moduli, as an orthotropic plate model of the building. According to these formulas, the reduced elastic moduli are less than the elastic modulus of the panels by approximately 8–15 times, and the reduced density of the plate model of the discrete part of the building is 7–12 times less than the density of the panel material. Such a discrepancy in modules is explained by the presence of a large number of voids and the cellular structure of the building.

For multi-story buildings, the following building dimensions are specified as initial data. The height and length of a multi-story building are taken to be equal to $b = nb_1$ and a, where *n* number of rooms across the width of one floor, b_1 – the size of one vertical transverse wall (the height of one floor of a multi-story building is considered constant); h_1 - thickness of external longitudinal vertical load-bearing walls; h_2 – thickness of internal interior transverse vertical walls; $h_{overlap}$ – floor thickness. To represent the values of the elasticity and density moduli, their values were calculated for the following dimensions of a multi-story building: $h_1 = 0.35m, \; h_2 = 0.20m, \; h_{\text{overlap}} = 0.2m, \; a_1 = 5m, \; b_1 = 3m, \; a = 30\,m.$

The height and length of a multi-storey building are taken to be equal, respectively. $b = nb_1$ and $a = 30\,m.$ Building width H varies. Using the initial data, the values of the coefficients of reduced elastic moduli were determined, shown in Table 1 for multi-story buildings, calculated using formulas (3).

TABLE 1. Coefficients for determining the elastic moduli of a continuous building model with given initial data.

Thickness	Coefficients and elastic characteristics of the building						
H (м)	ξ_0	ξ_{11}	ξ_{12}	ξ_{13}	ξ_{22}	ξ_{23}	ξ_{33}
15	0.100	0.103	0.070	0.067	0.138	0.04	0.102
18	0.099	0.092	0.058	0.067	0.127	0.04	0.102
20	0.098	0.086	0.053	0.067	0.121	0.04	0.102

The proposed spatial model of the building takes into account all types of deformation and stress components and, as shown in work [14], is suitable for spatial calculation of seismic resistance of buildings and structures.

STATEMENT OF THE PROBLEM

The problem of transverse oscillations of a multi-story building is an antisymmetric problem of the bimoment theory of plate structures developed in [13, 14]. Seismic oscillations of a multi-story building within the framework of the plate model are considered in a rectangular Cartesian coordinate system x_1, x_2 and z. For convenience, the origin of coordinates is located in the lower left corner of the middle surface of the continuous plate model of a multi-story building. Let's direct the axes OX_1 and OX_2 by length and height, and the axis OZ – along the width of a multi-story building (plate model).

The formulation of the problem of seismic resistance of multi-story buildings with transverse oscillations of a multi-story building is described within the framework of a plate model [14]. It should be noted that in the general case, transverse oscillations of a multi-story building are described using equations for shear forces and moments, as well as bimoments described by nine unknown kinematic functions:

$$\tilde{u}_k = \frac{u_k^{(+)} - u_k^{(-)}}{2}, \quad \tilde{\psi}_k = \frac{1}{2h^2}\int_{-h}^{h} u_k z dz, \quad \tilde{\beta}_k = \frac{1}{2h^4}\int_{-h}^{h} u_k z^3 dz, \; (k = 1,2),$$
$$\tilde{W} = \frac{u_3^{(+)} + u_3^{(-)}}{2}, \quad \tilde{r} = \frac{1}{2h}\int_{-h}^{h} u_3 dz, \quad \tilde{\gamma} = \frac{1}{2h^3}\int_{-h}^{h} u_3 z^2 dz. \tag{7}$$

When constructing the equation of transverse oscillations of multi-story buildings within the framework of a continuous plate model of a multi-story building in general form within the framework of three-dimensional elasticity theory, exact expressions for bending and torsional moments are determined $m_{11},\ m_{22},\ m_{12}$ from stress $\sigma_{11}, \sigma_{12}, \sigma_{22}$, according to the following formulas:

$$m_{11} = \frac{H}{2}\left(E_{11}\tilde{\varepsilon}_{11} + E_{12}\tilde{\varepsilon}_{22} - E_{13}\tilde{\varepsilon}_{33}\right),$$
$$m_{22} = \frac{H}{2}\left(E_{12}\tilde{\varepsilon}_{11} + E_{22}\tilde{\varepsilon}_{22} - E_{23}\tilde{\varepsilon}_{33}\right), \; m_{12} = m_{21} = G_{12}\frac{H}{2}\tilde{\varepsilon}_{12} \tag{8}$$

where

$$\tilde{\varepsilon}_{11} = \frac{\partial \tilde{\psi}_1}{\partial x_1}, \quad \tilde{\varepsilon}_{22} = \frac{\partial \tilde{\psi}_2}{\partial x_2}, \quad \tilde{\varepsilon}_{33} = \frac{2(\tilde{r} - \tilde{W})}{H}, \tilde{\varepsilon}_{12} = \frac{\partial \tilde{\psi}_1}{\partial x_2} + \frac{\partial \tilde{\psi}_2}{\partial x_1}.$$

The expressions of shear forces are determined from the tangential transverse stresses σ_{13}, σ_{23} by formulas:

$$Q_{13} = \int_{-h}^{h} \sigma_{13} dz = G_{13}\left(2\tilde{u}_1 + H\frac{\partial \tilde{r}}{\partial x_1}\right), \quad Q_{23} = \int_{-h}^{h} \sigma_{23} dz = G_{23}\left(2\tilde{u}_2 + H\frac{\partial \tilde{r}}{\partial x_2}\right). \tag{9}$$

Similarly, to describe the transverse oscillations of a multi-story building, in addition to moments and forces, bimoments are also introduced in general form. Within the framework of the three-dimensional theory of elasticity, exact expressions for normal and torsional bimoments from stresses are determined $\sigma_{11}, \sigma_{12}, \sigma_{22}$. Expressions of bending and torsional bimoments $p_{11},\ p_{22},\ p_{12}$ will be written in the form:

$$p_{11}=\frac{1}{2}\left(E_{11}\tilde{e}_{11}+E_{12}\tilde{e}_{22}-E_{13}\tilde{e}_{33}\right),$$
$$p_{12}=P_{21}=\frac{1}{2}G_{12}\tilde{e}_{12},\quad p_{22}=\frac{1}{2}\left(E_{12}\tilde{e}_{11}+E_{22}\tilde{e}_{22}-E_{23}\tilde{e}_{33}\right) \tag{10}$$

where $\tilde{e}_{11}=\frac{\partial\tilde{\beta}_1}{\partial x_1}$, $\tilde{e}_{12}=\frac{\partial\tilde{\beta}_2}{\partial x_2}$, $\tilde{e}_{33}=\frac{2(3\tilde{\gamma}-\tilde{W})}{H}$, $\tilde{e}_{12}=\frac{\partial\tilde{\beta}_1}{\partial x_2}+\frac{\partial\tilde{\beta}_2}{\partial x_1}$,

In addition to these values, three more specific transverse bimoments are introduced. Two of them are expressions for specific transverse bimoments $\tilde{p}_{13}$, $\tilde{p}_{23}$, which are constructed accurately using shear stresses σ_{13}, σ_{23} in the form of:

$$\tilde{p}_{13}=G_{13}\tilde{e}_{13},\quad \tilde{p}_{23}=G_{23}\tilde{e}_{23}. \tag{11}$$

where $\tilde{e}_{k3}=\left(\frac{2\tilde{u}_k-4\tilde{\psi}_k}{H}+\frac{\partial\tilde{\gamma}}{\partial x_k}\right)$, $(k=1,2)$.

And the third is the specific bimoment $\tilde{p}_{33}$ from normal stress σ_{33} in the form of:

$$\tilde{p}_{33}=E_{31}\tilde{\varepsilon}_{11}+E_{32}\tilde{\varepsilon}_{22}+E_{33}\tilde{\varepsilon}_{33}. \tag{12}$$

where $\tilde{\varepsilon}_{11}=\frac{\partial\tilde{\psi}_1}{\partial x_1}$, $\tilde{\varepsilon}_{22}=\frac{\partial\tilde{\psi}_2}{\partial x_1}$, $\tilde{\varepsilon}_{33}=-\frac{2(\tilde{r}-\tilde{W})}{H}$.

The equations of motion of transverse oscillations of a multi-story building within the framework of a plate model are described using a system of nine equations. We present the basic equations of transverse oscillations of a continuous plate model of multi-story buildings under seismic impacts, proposed in works [13, 14].

The first three equations of transverse oscillations of a plate model of a building are constructed with respect to bending, torque moments and shear forces:

$$\frac{\partial m_{11}}{\partial x_1}+\frac{\partial m_{12}}{\partial x_2}-\frac{m_{13}}{H}+\frac{\tilde{q}_1}{H}=\frac{1}{2}\rho\ddot{\tilde{\psi}}_1,$$
$$\frac{\partial m_{21}}{\partial x_1}+\frac{\partial m_{22}}{\partial x_2}-\frac{m_{23}}{H}+\frac{\tilde{q}_2}{H}=\frac{1}{2}\rho\ddot{\tilde{\psi}}_2, \tag{13}$$
$$\frac{\partial m_{13}}{\partial x_1}+\frac{\partial m_{23}}{\partial x_2}+\frac{2\tilde{q}_3}{H^2}=\frac{\rho\ddot{\tilde{r}}}{H}.$$

Within the framework of the plate model of the building how to show in [13, 14], three more equations of transverse oscillations of a multi-story building are derived, which are described relative to bimoments. Let us write down the equations of transverse oscillations of the plate model of the building, obtained relative to bending and torsional bimoments:

$$\frac{\partial p_{11}}{\partial x_1}+\frac{\partial p_{12}}{\partial x_2}-\frac{3\tilde{p}_{13}}{H}+\frac{\tilde{q}_1}{H}=\frac{1}{2}\rho\ddot{\tilde{\beta}}_1,$$
$$\frac{\partial p_{21}}{\partial x_1}+\frac{\partial p_{22}}{\partial x_2}-\frac{3\tilde{p}_{23}}{H}+\frac{\tilde{q}_2}{H}=\frac{1}{2}\rho\ddot{\tilde{\beta}}_2. \tag{14}$$

Now, let us present the equation of transverse oscillations of a plate model of a multi-story building with respect to specific tangential and longitudinal bimoments:

$$\frac{\partial\tilde{p}_{13}}{\partial x_1}+\frac{\partial\tilde{p}_{23}}{\partial x_2}-\frac{4\tilde{p}_{33}}{H}=\rho\ddot{\tilde{\gamma}}. \tag{15}$$

It should be noted that the presented six equations of transverse oscillations of a multi-story building within the framework of the bimoment theory of plate structures are described with respect to nine unknown generalized displacement functions $\tilde{\psi}_1$, $\tilde{\psi}_2$, $\tilde{u}_1$, $\tilde{u}_2$, $\tilde{\beta}_1$, $\tilde{\beta}_1$, $\tilde{r}$, $\tilde{\gamma}$, $\tilde{W}$: (7) - (15).

Using the method of expansion of displacements in an infinite Maclaurin series, three more equations of transverse oscillations of a plate model of a multi-story building with respect to generalized displacement functions were constructed in [13, 14] $\tilde{u}_1$, $\tilde{u}_2$, $\tilde{W}$ points of the external walls in the following form:

$$\tilde{u}_1=\frac{1}{2}\left(21\tilde{\beta}_1-7\tilde{\psi}_1\right)-\frac{1}{30}H\frac{\partial\tilde{W}}{\partial x_1},$$
$$\tilde{u}_2=\frac{1}{2}\left(21\tilde{\beta}_2-7\tilde{\psi}_2\right)-\frac{1}{30}H\frac{\partial\tilde{W}}{\partial x_2}, \quad (16)$$
$$\tilde{W}=\frac{1}{4}\left(21\tilde{\gamma}-3\tilde{r}\right)-\frac{1}{20}H\left(\frac{E_{31}}{E_{33}}\frac{\partial\tilde{u}_1}{\partial x_1}+\frac{E_{32}}{E_{33}}\frac{\partial\tilde{u}_2}{\partial x_2}\right).$$

When describing the boundary conditions for the equations of transverse oscillations of buildings (7) - (12), in addition to forces, moments and bimoments, we also introduce specific bimoments $\tilde{\sigma}_{11},\ \tilde{\sigma}_{22}, \tilde{\sigma}_{12},\ \tilde{\sigma}^*_{11},\ \tilde{\sigma}^*_{22}$, which are determined by the formulas obtained in [13-16], in the form:

$$\tilde{\sigma}_{11}=\left(E_{11}-\frac{E_{13}}{E_{33}}E_{31}\right)\frac{\partial\tilde{u}_1}{\partial x_1}+\left(E_{12}-\frac{E_{13}}{E_{33}}E_{32}\right)\frac{\partial\tilde{u}_2}{\partial x_2},$$
$$\tilde{\sigma}_{22}=\left(E_{21}-\frac{E_{23}}{E_{33}}E_{31}\right)\frac{\partial\tilde{u}_1}{\partial x_1}+\left(E_{22}-\frac{E_{23}}{E_{33}}E_{32}\right)\frac{\partial\tilde{u}_2}{\partial x_2},\ \tilde{\sigma}_{12}=G_{12}\left(\frac{\partial\tilde{u}_1}{\partial x_2}+\frac{\partial\tilde{u}_2}{\partial x_1}\right). \quad (17)$$

Specific bimoments $\tilde{\sigma}^*_{11},\ \tilde{\sigma}^*_{22}$ will be written in the form:

$$\tilde{\sigma}^*_{11}=-E_{11}\frac{\partial^2\tilde{W}}{\partial x_1^2}-E_{12}\frac{\partial^2\tilde{W}}{\partial x_1^2}+E_{13}\frac{60\left(3\tilde{W}+4\tilde{r}-21\tilde{\gamma}\right)}{H},$$
$$\tilde{\sigma}^*_{22}=-E_{12}\frac{\partial^2\tilde{W}}{\partial x_1^2}-E_{22}\frac{\partial^2\tilde{W}}{\partial x_1^2}+E_{23}\frac{60\left(3\tilde{W}+4\tilde{r}-21\tilde{\gamma}\right)}{H}, \quad (18)$$

When considering transverse oscillations of a building, the maximum stresses between longitudinal and transverse walls are determined by the formulas:

$$\tilde{\sigma}^*_{13}=G_{13}\left[\frac{7}{6}\frac{\partial\tilde{W}}{\partial x_1}+\frac{E_{31}}{E_{33}}\frac{H}{36}\frac{\partial^2\tilde{u}_1}{\partial x_1^2}+\frac{H}{36}\frac{E_{32}}{E_{33}}\frac{\partial^2\tilde{u}_2}{\partial x_1\partial x_2}-\frac{35\left(33\tilde{\beta}_1-9\tilde{\psi}_1-4\tilde{u}_1\right)}{6H}\right]-\frac{1}{6}\tilde{q}_1-\frac{G_{31}}{E_{33}}\frac{H}{36}\frac{\partial\tilde{q}_3}{\partial x_1},$$
$$\tilde{\sigma}^*_{23}=G_{23}\left[\frac{7}{6}\frac{\partial\tilde{W}}{\partial x_2}+\frac{E_{31}}{E_{33}}\frac{H}{36}\frac{\partial^2\tilde{u}_1}{\partial x_2\partial x_1}+\frac{H}{36}\frac{E_{32}}{E_{33}}\frac{\partial^2\tilde{u}_2}{\partial x_2^2}-\frac{35\left(33\tilde{\beta}_2-9\tilde{\psi}_2-4\tilde{u}_2\right)}{6H}\right]-\frac{1}{6}\tilde{q}_2-\frac{G_{32}}{E_{33}}\frac{H}{36}\frac{\partial\tilde{q}_3}{\partial x_2}, \quad (19)$$

$$\tilde{\sigma}^*_{33}=\frac{5E_{31}}{6}\frac{\partial\tilde{u}_1}{\partial x_1}+\frac{5E_{32}}{6}\frac{\partial\tilde{u}_2}{\partial x_2}+\frac{H}{36}\left(E_{31}\frac{\partial^2\tilde{W}}{\partial x_1^2}-E_{32}\frac{\partial^2\tilde{W}}{\partial x_2^2}\right)-E_{33}\frac{35\left(9\tilde{\gamma}-2\tilde{W}-\tilde{r}\right)}{6H}+$$
$$+\frac{1}{6}\tilde{q}_3-\frac{H}{36}\left(E_{31}\frac{\partial}{\partial x_1}\left(\frac{\tilde{q}_1}{G_{13}}\right)+E_{32}\frac{\partial}{\partial x_2}\left(\frac{\tilde{q}_2}{G_{23}}\right)\right) \quad (20)$$

Next, we define the boundary conditions for the problem under consideration on transverse oscillations of multi-story buildings.

On the free lateral faces of the building we have the conditions of equality to zero of forces, moments and bimoments and force factors:

$$m_{11}=0,\ m_{12}=0,\ \ p_{11}=0,\ p_{12}=0,\ \ m_{13}=0,\ \ \tilde{p}_{13}=0,\ \tilde{\sigma}_{11}=0,\ \ \tilde{\sigma}_{12}=0,\ \ \sigma^*_{13}=0. \quad (21, a)$$

On the free upper edge of the building we have the following conditions:

$$m_{22}=0,\ m_{12}=0,\ \ p_{22}=0,\ p_{12}=0,\ \ m_{23}=0,\ \ \tilde{p}_{23}=0,\ \tilde{\sigma}_{22}=0,\ \ \tilde{\sigma}_{12}=0,\ \ \sigma^*_{23}=0. \quad (21, b)$$

On the side faces of the building located in the ground we have conditions of equality of force factors and loads from the ground:

$$m_{11}=m^{\text{gr}}_{11},\ \ m_{12}=m^{\text{gr}}_{12},\ \ \tilde{p}_{11}=p^{\text{gr}}_{11},\ \ \tilde{p}_{12}=p^{\text{gr}}_{12},\ \ \tilde{p}_{13}=p^{\text{gr}}_{13},\ \ \tilde{\tau}_{13}=\tau^{\text{gr}}_{12},$$
$$\tilde{\sigma}_{11}=\sigma^{\text{gr}}_{12};\ \ \tilde{\sigma}_{12}=\sigma^{\text{gr}}_{12}\ \ \tilde{\sigma}^*_{13}=\sigma^{\text{gr}}_{12}. \quad (22, a)$$

On the free upper edge of the building we have the conditions:

$$m_{12} = m_{11}^{gr},\ \ m_{22} = m_{12}^{gr},\ \ \tilde{p}_{12} = p_{12}^{gr},\ \ \tilde{p}_{22} = p_{22}^{gr},\ \ \tilde{p}_{23} = p_{23}^{gr},\ \ \tilde{\tau}_{23} = \tau_{23}^{gr},$$
$$\tilde{\sigma}_{22} = \sigma_{22}^{gr};\ \ \tilde{\sigma}_{12} = \sigma_{12}^{gr}\ \ \tilde{\sigma}_{23}^{*} = \sigma_{23}^{gr}. \qquad (22,\text{a})$$

The methodology and algorithm for the numerical solution of the problem of oscillations of a multi-story building with transverse oscillations are developed based on the finite difference method. To approximate the derivatives of displacements by spatial coordinates, we use the formulas of central difference schemes.

To approximate the derivatives of stresses, forces, moments and bimoments, central finite difference schemes on half-steps are used, which have the second order of accuracy. The conditions for the equality to zero of the force factors of a multi-story building at free edges are approximated as the equality to zero of the arithmetic mean value of the displacements of external and internal points. The program for calculating the displacements and force factors of a multi-story building is compiled in the Delphi algorithmic environment.

ANALYSIS OF NUMERICAL RESULTS

Numerical calculations are made under the assumption that seismic soil movement occurs in the direction of the *OZ* axis (along the width of the building) in the form of acceleration of the building base:

$$\ddot{u}_0(t) = a_0 \cos(\omega_0 t) \qquad (23)$$

where $a_0 = k_c g$ - maximum acceleration and $\omega_0 = 2\pi\nu_0$ – circular frequency of the soil base, k_c и ν_0 - the earthquake intensity coefficient and the natural frequency of the external impact, respectively.

From here we obtain the displacement of the building foundation in the forme:

$$u_0(t) = \frac{A_0}{2}(1 - \cos(\omega_0 t)). \qquad (24)$$

here $A_0 = \dfrac{2k_c g}{\omega_0^2}$. – amplitude of the base displacement. We take zero values as the initial conditions. Note that the seismicity coefficients for seven-point, eight-point and nine-point earthquakes are equal to $k_c = 0.1,\ 0.2,\ 0.4$, respectively.

We assume that the external walls are made of reinforced concrete with a modulus of elasticity $E_0 = 30000\ MPa$ density $\rho_0 = 2500$ kg/m^3, Poisson's ratio $\nu_0 = 0.3$. Внутренние We consider the walls to be made of expanded clay concrete with the following physical characteristics: modulus of elasticity $E = 7500\,MPa$, density $\rho = 1200\ kg/\mathrm{m}^3$, Poisson's ratio $\nu = 0.3$. The foundation of the building consists of reinforced concrete with a modulus of elasticity $E_{Found} = 2500\ MPa$, density $\rho_{Found} = 2500\ kg/\mathrm{m}^3$, Poisson's ratio $\nu_{Found} = 0.3$. Foundation width $h_{Found} = 1.2\ m$. The results of calculations of forced oscillations of a building within the framework of a plate structure model are presented for the following dimensions of the building slabs: $h_1 = 0.30m,\ h_2 = 0.20m,\ h_{\text{overlap}} = 0.2m,\ a_1 = 5m,\ b_1 = 3m.$

The height and length of a multi-storey building are taken to be equal, respectively. $b = nb_1$ and $a = 30\,м,$ and the width of the building H varies. Using the initial data, the values of the reduced elasticity, shear and density moduli presented in Table 1 of multi-story buildings were determined, calculated using the formulas given in [15-17]. With transverse oscillations of the displacement distribution field $\tilde{u}_2, \tilde{\psi}_2, \tilde{\beta}_2, \tilde{W}, \tilde{r},\ \tilde{\gamma}$, arising under the action of seismic load are symmetrical. Similarly, the stresses $\tilde{\sigma}_{11},\ \tilde{\sigma}_{22}$, moments m_{11} , m_{22} and bimoments p_{11} , p_{22}, $\tilde{p}_{33}$, $\tilde{\tau}_{33}$ are symmetrical, which indicates the reliability of the results obtained.

We present the numerical results obtained using the developed methodology and algorithm for calculating multi-story buildings under longitudinal seismic impacts. Note that the maximum stress values are determined in the external walls of a large-panel multi-story building. The amplitude of the external impact A_0 depends on the magnitude of the earthquake, which is determined from the condition $A_0\omega_0^2 = k_c g$, where $k_c,\ g$ - seismicity coefficient and acceleration of gravity. Then the amplitude of the external impact will be equal to $A_0 = k_c g / \omega_0^2$.

Table 2 shows the maximum values of normal stresses. σ_{22} in MPa measurements obtained in the contact zone of vertical walls of 20-, 24- and 28-story buildings during transverse oscillations under seismic impact with an intensity of seven points

TABLE 2. Maximum values of normal stress σ_{22} , in MPa, for high-rise buildings with a width of H=18 m with transverse oscillations depending on the frequency of a seven-point external impact.

H, m	ν_0, *Hz*	**20- storey building**	**24- storey building**	**28- storey building**
	2.6	5,339	5,589	7,615
	2.8	4,941	5,489	8,948
18	3.0	4,859	5,091	9,882
	3.2	4,744	6,233	13,80
	3.4	5,059	6,179	14,49

Calculations were performed for five values of external influence frequency: $\nu_0 = 2{,}6;\ 2{,}8;\ 3{,}0;\ 3{,}2;\ 3{,}4\ Hz$ which correspond to seismic impacts during earthquakes on the territory of our republic.

Maximum value of normal stress σ_{22} in the lower part of the external wall of a 20-storey building in the frequency range from 2.6 to 3.4 *Hz* under seismic impacts of seven points takes small values within the range from $\sigma_{22} = 4.7\ MPa$ to $\sigma_{22} = 5.3\ MPa$ (Table 2).

Maximum value of normal stress σ_{22} in the lower part of the external wall of a 24-storey building in the frequency range from 2.6 to 3.4 *Hz* under seismic impacts of magnitude 7 also takes small values within the range from $\sigma_{22} = 5.6 MPa$ to $\sigma_{22} = 6.2 MPa$. As can be seen, with a seven-point seismic impact with an external impact frequency $\nu_0 = 3{,}4\ Hz$, The maximum stress value is significant and amounts to $\sigma_{22} = 6.2\ MPa$ (Table 2).

Maximum value of normal stress σ_{22} in the lower part of the external wall of a 28-storey building in the frequency range from 2.6 to 3.4 *Hz* under seven-point seismic impacts takes on small values within the range from $\sigma_{22} = 7.6 MPa$ to $\sigma_{22} = 15\ MPa$ (Table 2). As we can see, in the external load-bearing walls of a 28-storey building, with the frequency value of external influence $\nu_0 = 3.0 - 3{,}4\ Hz$, the maximum stress value is much higher and can reach $\sigma_{22} = 15 MPa$. Table 3 presents the results of calculations for the maximum values of horizontal displacement in *cm*, along the width (small size), obtained at the upper levels of 20-, 24- and 28-story buildings with a width of H=18 m under transverse oscillations under seismic action with an intensity of seven points.

TABLE 3. Maximum values of normal horizontal displacement $\tilde{r}$ (in cm) for a high-rise building with a width of *H=18 m* with transverse oscillations.

H, m	ν_0, *Hz*	**20- storey building**	**24- storey building**	**28- storey building**
	2.6	5,301	5,742	6,653
	2.8	5,066	5,340	7,887
18	3.0	5,166	5,563	9,081
	3.2	4,909	5,273	17,62
	3.4	5,059	6,179	14,49

The calculations were performed for five values of the frequency of external action $\nu_0 = 2{,}6;\ 2{,}8;\ 3{,}0;\ 3{,}2;\ 3{,}4\ Hz$.

The maximum values of normal horizontal displacement are calculated $\tilde{r}$ (in *cm*) for a 20-storey high-rise building with a width of *H=18 m* with transverse oscillations at the upper level of the 20th floor in the frequency range from 2.6 to 3.4 *Hz* under seven-point seismic impacts. The horizontal displacement of the building takes on small values within the range from $\tilde{r} = 4.9\ cm$ to $\tilde{r} = 5.3 cm$ (Table 3).

Maximum horizontal displacement values found $\tilde{r}$ (in *cm*) for a 24-storey high-rise building with a width of H=18 m with transverse oscillations at the upper level of the 24th floor in the frequency range from 2.6 to 3.4 *Hz* under seismic impacts of seven points. The horizontal displacement of the building takes small values within the range from $\tilde{r} = 4.74 cm$ to $\tilde{r} = 6.22 cm$ (Table 3).

CONCLUSION

Thus, a continuum model, a method and an algorithm for numerical solution of the seismic resistance problem, as well as methods for determining displacements, stresses and forces during transverse oscillations of a multi-story building are proposed. With an increase in the height of the floor, normal stresses increase significantly σ_{22}, horizontal movements $\tilde{r}$. The analysis of numerical results showed that the stress-strain state of a multi-story building depends significantly on the values of the floor height a_1 and thickness of external walls. Based on the analysis of numerical results, it was found that the plate model is suitable for describing the dynamic behavior and calculating the stress-strain state of multi-story buildings under seismic impacts.

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