

# Advanced Study Workshop on Earthquake Engineering

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## Transverse vibrations of the support with piecewise homogeneous of the soil spring contacts

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# Transverse vibrations of the support with piecewise homogeneous of the soil spring contacts

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**Abstract.** The behavior of supports plays a major role in the operation of highway bridges during earthquakes. For many highway bridges, the piers absorb most of the seismic force, especially in the longitudinal direction. The performance of a bridge structure during an earthquake is likely to depend on the proximity of the bridge to the fault and the site conditions. Both of these factors influence the intensity of ground shaking and ground deformation, as well as the variability of these effects along the length of the bridge. This paper examines the influence of foundation depth on the stress state of two bridge supports in the form of piles under kinematic excitation of the foundation.

## INTRODUCTION

Various bridges structures, including bridges, are of great importance worldwide for expanding trunk road networks, increasing the volume of passenger and cargo traffic, and developing the infrastructure of large cities. Bridge structures, being one of the types of construction objects, have specific consumer properties that determine their purpose and quality. Due to population growth and urban development, the construction of highways in cities has increased significantly. Bridge structures are an important part of road networks and play an important role in reducing traffic congestion, keeping road bridge structures running in the event of a disaster such as an earthquake. Bridge structures and other structures in seismic regions must be able to withstand the forces of earthquakes in any direction. [1-2].

Issues on improving the methodology for calculating trans port structures, in particular bridges, for seismic resistance are currently acquiring special relevance in connection with the widespread development of construction in seismic areas [3].

Earthquake damage to a bridge structures can have severe consequences. Clearly, the collapse of a bridges places people on or below the bridge structures at risk, and it must be replaced after the earthquake unless alternative transportation paths are identified. The consequences of less severe damage are less obvious and dramatic, but they are nonetheless important. After the 1971 San Fernando earthquake, it became quite evident that many abutments had been subjected to large seismic forces. On many bridges, abutment damage was the only damage reported indicating that abutments attracted a large portion of the seismic force. Soil-abutment interaction under seismic loads is a highly non-linear phenomenon [4-5].

During the 1995 Hyogo-Ken Nanbu (Kobe) earthquake, significant damage and collapse likewise occurred in elevated roadways and bridges founded adjacent to or within Osaka Bay. Several types of site conditions contributed to the failures. First, many of the bridges were founded on sand-gravel terraces (alluvial deposits) overlying gravel-sand-mud deposits at depths of less than 33 ft (10 m), a condition which is believed to have led to site amplification of the bedrock motions. Furthermore, many of the sites were subject to liquefaction and lateral spreading, resulting in

permanent substructure deformations and loss of superstructure support (Fig.1). Finally, the site was directly above the fault rupture, resulting in ground motions having high horizontal and vertical ground accelerations as well as large velocity pulses. Near-fault ground motions can impose large deformation demands on yielding structures, as was evident in the overturning collapse of all 17 bents of the Higashi-Nada Viaduct of the Hanshin Expressway, Route 3, in Kobe (Fig.2). Other factors contributed to the behavior of structures in Kobe [6-7].



**FIGURE 1.** Nishinomiya-ko Bridge approach span collapse in the 1995 Hyogo-Ken Nanbu earthquake (Kobe Collection, EERC Library, University of California, Berkeley) [4-7]



**FIGURE 2.** Higashi-Nada Viaduct collapse in the 1995 Hyogo-Ken Nanbu earthquake. (Source: EERI, The HyogoRen Nambu Earthquake, January 17, 1995, Preliminary Reconnaissance Report, Feb. 1995.) [4-7]

Soil-structure interaction is of great significance in seismic design and assessment of reinforced concrete (RC) bridge structures. Recent research studies indicate that foundation-pile interaction effects may considerably alter the seismic response of R.C. bridges by partly mitigating the deformation demands in the piers at the expense of some permanent residual deformation of the foundation [8]. The seismic analysis carried out assuming foundation to be perfectly rigid and bonded to the soil underneath is far from truth and therefore, the soil-pile interaction effect on the dynamic behavior of the bridge pier should be considered [9]. Soil-structure interaction is now a common term to discuss among the research fraternity, where the response of the soil influences the motion in the structure and vice-versa [10]. When an external force such as earthquake act on the system structural displacement and ground displacement will be developing and are mutually dependent on each other. The process of response of soil influence the motion of the structure and vice versa is termed as soil structure interaction. Soil Structure Interaction effect is important in design of engineering structures especially structures founded on soft and medium soil. In hard soil the effect is negligible. And also Winkler spring model is more popular and studies are mainly concentrated on this method. So this work is attempted to study the effect of SSI on seismic response of bridges with pile foundation situated in Indian soil condition by using Winkler method. Assumptions used in this study are: 1. The effect of abutments are neglected for interior spans. 2. Piers are of equal height [11].

## METHODOLOGY

To assess the reliability and bearing capacity of road girder bridges on supports, in addition to the moving loads acting on the girder, we should take into account the forces transmitted through the girder supports related to the

impact of, for example, seismic waves [12]. The aim of this article is to study the transverse vibrations of a single-span beam reinforced concrete bridge, the pile part of which, under seismic action, interacts with the surrounding soil. When calculating the bearing capacity and settlement of single piles, preference should be given to tabulated or analytical solutions given in building regulations. When designing pile foundations, the following factors should be considered: soil conditions at the construction site; hydrogeological regime; pile installation features; and the presence of slurry under the lower end of the piles. All calculations of piles, pile foundations, and their bases should be performed using estimated values for material and soil characteristics.

This paper examines the vibrations of two square beam piles of equal length  $L$ , partially embedded in soil, and a supporting rigid beam of mass  $M$ . Each pile interacts with the soil according to Winkler's law (Fig.3). The foundation undergoes horizontal motion according to the law  $W=W_0\sin\omega t$ .

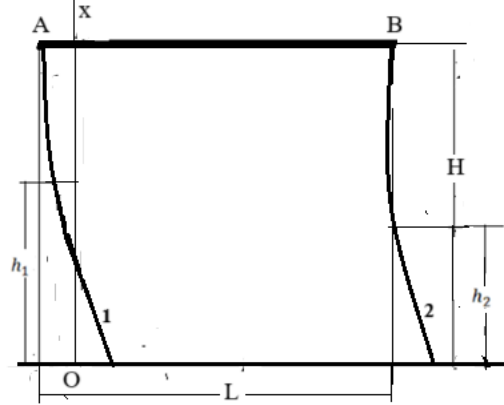


FIGURE 3. Diagram of oscillation of two piles supporting a rigid beam

We will set the origin of coordinates in the lower section of the first pile and direct the OX axis vertically upward. We denote the deflections of each pile by  $w_1(x,t)$  and  $w_2(x,t)$ . In this case, we assume

$$w_1(x,t)=w_{11}(x,t) \text{ for } 0 < x < h_1, \quad w_1(x,t)=w_{12}(x,t) \text{ for } h_1 < x < H$$

$$w_2(x,t)=w_{21}(x,t) \text{ for } 0 < x < h_2, \quad w_2(x,t)=w_{22}(x,t) \text{ for } h_2 < x < H,$$

where  $h_1$  and  $h_2$  are the lengths of each pile driven into the ground, and  $H$  is their total length. In each zone, the deflections  $w_{ij}(x,t)$  satisfy the equations

$$EJ \frac{\partial^4 w_{11}}{\partial x^4} + m \frac{\partial^2 w_{11}}{\partial t^2} + k_1 w_{11} = 0 \text{ for } 0 < x < h_1 \quad (1)$$

$$EJ \frac{\partial^4 w_{12}}{\partial x^4} + m \frac{\partial^2 w_{12}}{\partial t^2} = 0 \text{ for } h_1 < x < H \quad (2)$$

$$EJ \frac{\partial^4 w_{21}}{\partial x^4} + m \frac{\partial^2 w_{21}}{\partial t^2} + k_2 w_{21} = 0 \text{ for } 0 < x < h_2 \quad (3)$$

$$EJ \frac{\partial^4 w_{22}}{\partial x^4} + m \frac{\partial^2 w_{22}}{\partial t^2} = 0 \text{ for } h_2 < x < H \quad (4)$$

The functions  $w_{ij}(x,t)$  satisfy zero initial and the following boundary conditions

$$\frac{\partial w_{11}}{\partial x} = 0, \quad EJ \frac{\partial^3 w_{11}}{\partial x^3} = k_0 (w_{11} - w_0) \text{ for } x=0 \quad (5)$$

$$w_{11} = w_{12}, \quad \frac{\partial w_{11}}{\partial x} = \frac{\partial w_{12}}{\partial x}, \quad \frac{\partial^2 w_{11}}{\partial x^2} = \frac{\partial^2 w_{12}}{\partial x^2}, \quad \frac{\partial^3 w_{11}}{\partial x^3} = \frac{\partial^3 w_{12}}{\partial x^3} \text{ for } x=h_1 \quad (6)$$

$$\frac{\partial w_{21}}{\partial x} = 0, \quad EJ \frac{\partial^3 w_{21}}{\partial x^3} = k_0 (w_{21} - w_0) \text{ for } x=0 \quad (7)$$

$$w_{21} = w_{22}, \quad \frac{\partial w_{21}}{\partial x} = \frac{\partial w_{22}}{\partial x}, \quad \frac{\partial^2 w_{21}}{\partial x^2} = \frac{\partial^2 w_{22}}{\partial x^2}, \quad \frac{\partial^3 w_{21}}{\partial x^3} = \frac{\partial^3 w_{22}}{\partial x^3}, \text{ for } x=h_2 \quad (8)$$

$$\frac{\partial w_{12}}{\partial x} = 0, \frac{\partial w_{22}}{\partial x} = 0, w_{12} = w_{22} = w(t), \frac{\partial^3 w_{12}}{\partial x^3} + \frac{\partial^3 w_{22}}{\partial x^3} = \frac{M}{EJ} \frac{\partial^2 w_{22}}{\partial t^3} \text{ for } x=H \quad (9)$$

where  $EJ$  is the bending rigidity of the beam,  $m$  is the linear mass,  $k_1, k_2$  are the bedding factors for each contact zone of the pile (beam) with the ground, respectively,  $k_0$  is the contact rigidity factor for shear of the lower section of the beams with the base,  $w(t)$  is the displacement of the rigid beam.

The solution of equations (1) - (4) and the beam oscillation amplitude  $w(t)$  are represented in the form

$$w_{ij}(x,t) = W_{ij}(x) \sin \omega_0 t, \quad w = W \sin \omega_0 t,$$

where the functions  $C_{ij}(x)$  satisfy the equations and, according to (5) - (8), the following conditions

$$EJ \frac{\partial^4 w_{11}}{\partial x^4} + (k_1 - m\omega_0^2) W_{11} = 0 \text{ for } 0 < x < h_1 \quad (10)$$

$$EJ \frac{\partial^4 w_{12}}{\partial x^4} - m\omega_0^2 W_{12} = 0 \text{ for } h_1 < x < H \quad (11)$$

$$EJ \frac{\partial^4 w_{21}}{\partial x^4} + (k_2 - m\omega_0^2) W_{21} = 0 \text{ for } 0 < x < h_2 \quad (12)$$

$$EJ \frac{\partial^4 w_{22}}{\partial x^4} - m\omega_0^2 W_{22} = 0 \text{ for } h_2 < x < H \quad (13)$$

$$W'_{11} = 0, W''_{11} = k_0(W_{11} - W_0) \text{ for } x=0 \quad (14)$$

$$W_{11} = W_{12}, W'_{11} = W'_{12}, W''_{11} = W''_{12}, W'''_{11} = W'''_{12}, \text{ for } x=h_1 \quad (15)$$

$$W'_{12} = 0, W''_{12} = k_0(W_{12} - W_0) \text{ for } x=0 \quad (16)$$

$$W_{21} = W_{22}, W'_{21} = W'_{22}, W''_{21} = W''_{22}, W'''_{21} = W'''_{22}, \text{ for } x=h_2 \quad (17)$$

$$W'_{12} = 0, W'_{22} = 0, W_{12} = W, W_{22} = W, \text{ for } x=H \quad (18)$$

Let us consider the case of rigid contact between the beam and the piles, i.e., we assume that  $k_0 \rightarrow \infty$ . In this case, the equation of beam motion (9) is written as

$$W''_{12} + W''_{22} = -W \frac{M}{EJ} \omega_0^2.$$

Assuming that  $k_2 < k_1$ ,  $\omega_0 < \sqrt{k_2/4m}$ , we represent the solution of equations (9)-(12), satisfying conditions (15) and (17), through Krylov functions

$$W_{11} = C_1 Y_1(\beta_1(\xi - \bar{h}_1)) + \frac{C_2 \omega Y_2(\beta_1(\xi - \bar{h}_1))}{2\beta_1 b} + C_3 \omega^2 Y_3(\beta_1(\xi - \bar{h}_1)) / 2\beta_1^2 + C_4 \omega^3 Y_4(\beta_1(\xi - \bar{h}_1)) / 4\beta_1^3,$$

$$W_{12} = C_1 S_1[\omega(\xi - \bar{h}_1)] + C_2 S_2[\omega(\xi - \bar{h}_1)] + C_3 S_3[\omega(\xi - \bar{h}_1)] + C_4 S_4[\omega(\xi - \bar{h}_1)],$$

$$W_{21} = B_1 Y_1(\beta_2(\xi - \bar{h}_2)) + \frac{B_2 \omega Y_2(\beta_2(\xi - \bar{h}_2))}{2\beta_2} + B_3 \omega^2 Y_3(\beta_2(\xi - \bar{h}_2)) / 2\beta_2^2 + B_4 \omega^3 Y_4(\beta_2(\xi - \bar{h}_2)) / 4\beta_2^3,$$

$$W_{22} = B_1 S_1[\omega(\xi - \bar{h}_2)] + B_2 S_2[\omega(\xi - \bar{h}_2)] + B_3 S_3[\omega(\xi - \bar{h}_2)] + B_4 S_4[\omega(\xi - \bar{h}_2)],$$

where  $Y_j(z)$  and  $S_j(z)$  are Krylov functions.

$$Y_1 = \cos \beta_1 z \operatorname{ch} \beta_1 z, \quad Y_2 = (\sin \beta_1 z \operatorname{ch} \beta_1 z + \cos \beta_1 z \operatorname{sh} \beta_1 z),$$

$$Y_3 = \sin \beta_1 z \operatorname{sh} \beta_1 z, \quad Y_4 = (\sin \beta_1 z \operatorname{ch} \beta_1 z - \cos \beta_1 z \operatorname{sh} \beta_1 z),$$

$$S_1 = 0.5 * (\operatorname{ch} \omega z + \cos \omega z), \quad S_2 = 0.5 * (\operatorname{sh} \omega z + \sin \omega z),$$

$$S_3 = 0.5 * (\operatorname{ch} \omega z - \cos \omega z), \quad S_4 = 0.5 * (\operatorname{sh} \omega z - \sin \omega z),$$

$$\xi = \frac{x}{H}, \quad \omega = \sqrt{\frac{mL^4 \omega_0^2}{EJ}}, \quad \beta_1 = \sqrt[4]{\omega_1^2 - \omega^2}, \quad \beta_2 = \sqrt[4]{\omega_2^2 - \omega^2}, \quad \omega_1 = \sqrt{\frac{k_1 L^4}{4EJ}}, \quad \omega_2 = \sqrt{\frac{k_2 L^4}{4EJ}}, \quad \bar{h}_1 = \frac{h_1}{H}, \quad \bar{h}_2 = \frac{h_2}{H}.$$

The constants  $C_i, B_i$  ( $i=1,2,3,4$ ) are determined from the boundary conditions (14, 16, 18), which give

$$c_{11}C_1 + c_{12}C_2 + c_{13}C_3 + c_{14}C_4 = W_0, \quad (19)$$

$$c_{12}C_1 + c_{22}C_2 + c_{23}C_3 + c_{24}C_4 = 0, \quad (20)$$

$$b_{11}B_1 + b_{12}B_2 + b_{13}B_3 + b_{14}B_4 = W_0, \quad (21)$$

$$b_{21}B_1 + b_{22}B_2 + b_{23}B_3 + b_{24}B_4 = 0, \quad (22)$$

$$a_{11}C_1 + a_{12}C_2 + a_{13}C_3 + a_{14}C_4 = W, \quad (23)$$

$$a_{21}C_1 + a_{22}C_2 + a_{23}C_3 + a_{24}C_4 = 0, \quad (24)$$

$$d_{11}B_1 + d_{12}B_2 + d_{13}B_3 + d_{14}B_4 = W, \quad (25)$$

$$d_{21}B_1 + d_{22}B_2 + d_{23}B_3 + d_{24}B_4 = 0, \quad (26)$$

where

$$\begin{aligned} c_{11} &= Y_1(-\beta_1 \bar{h}_1), c_{12} = Y_2(-\beta_1 \bar{h}_1) \omega / 2\beta_1, c_{13} = Y_3(-\beta_1 \bar{h}_1) \omega^2 / 2\beta_1^2, c_{14} = Y_4(-\beta_1 \bar{h}_1) \omega^3 / 4\beta_1^3, \\ c_{21} &= -\beta_1 Y_4(-\beta_1 \bar{h}_1), c_{22} = \omega Y_1(-\beta_1 \bar{h}_1), c_{23} = Y_2(-\beta_1 \bar{h}_1) \omega^2 / 2\beta_1, c_{24} = Y_3(-\beta_1 \bar{h}_1) \omega^3 / 2\beta_1, \\ b_{11} &= Y_1(-\beta_2 \bar{h}_2), b_{12} = \omega Y_2(-\beta_2 \bar{h}_2) \omega / 2\beta_2, b_{13} = Y_3(-\beta_2 \bar{h}_2) \omega^2 / 2\beta_2^2, b_{14} = Y_4(-\beta_2 \bar{h}_2) \omega^3 / 4\beta_2^3, \\ b_{21} &= -\beta_2 Y_4(-\beta_2 \bar{h}_2), b_{22} = \omega Y_1(-\beta_2 \bar{h}_2), b_{23} = Y_2(-\beta_2 \bar{h}_2) \omega^2 / 2\beta_2, b_{24} = Y_3(-\beta_2 \bar{h}_2) \omega^3 / 2\beta_2, \\ a_{11} &= S_1(\omega \bar{l}_1), a_{12} = S_2(\omega \bar{l}_1), a_{13} = S_3(\omega \bar{l}_1), a_{14} = S_4(\omega \bar{l}_1), \\ a_{21} &= S_4(\omega \bar{l}_1), a_{22} = S_1(\omega \bar{l}_1), a_{23} = S_2(\omega \bar{l}_1), a_{24} = S_3(\omega \bar{l}_1), \\ d_{11} &= S_1(\omega \bar{l}_2), d_{12} = S_2(\omega \bar{l}_2), d_{13} = S_3(\omega \bar{l}_2), d_{14} = S_4(\omega \bar{l}_2), \\ d_{21} &= S_4(\omega \bar{l}_2), d_{22} = S_1(\omega \bar{l}_2), d_{23} = S_2(\omega \bar{l}_2), d_{24} = S_3(\omega \bar{l}_2) \end{aligned}$$

where  $\bar{l}_1 = 1 - \bar{h}_1, \bar{l}_2 = 1 - \bar{h}_2, \bar{h}_1 = h_1/H, \bar{h}_2 = h_2/H$ .

We believe that

$$C_i = r_i W + s_i W_0, B_i = q_i W + n_i W_0, \quad (27)$$

where  $r_i, s_i, q_i$  and  $n_i$  are determined from the system (19) - (26)

$$\begin{aligned} r_1 &= \frac{P_2}{\Delta_1}, r_2 = -\frac{P_1}{\Delta_1}, r_3 = r_1 F_1 + r_2 F_2, r_4 = r_1 R_1 + r_2 R_2, \\ s_1 &= [(a_{23}F_0 + a_{24}R_0)N_2 - (a_{13}F_0 + a_{14}R_0)P_2] / \Delta_1, \\ s_2 &= [(a_{13}F_0 + a_{14}R_0)P_2 - (a_{23}F_0 + a_{24}R_0)N_1] / \Delta_1, \\ s_3 &= s_1 F_1 + s_2 F_2, s_4 = s_1 R_1 + s_2 R_2, \Delta_1 = N_1 P_2 - N_2 P_1, \\ F_1 &= (c_{14}c_{21} - c_{24}c_{11}) / \Delta_0, F_2 = (c_{14}c_{22} - c_{24}c_{12}) / \Delta_0, F_0 = c_{24} / \Delta_0, \\ R_1 &= (c_{23}c_{11} - c_{13}c_{21}) / \Delta_0, R_2 = (c_{23}c_{12} - c_{13}c_{22}) / \Delta_0, R_0 = -c_{23} / \Delta_0, \\ N_1 &= a_{12} + a_{13}F_1 + a_{14}R_1, N_2 = a_{11} + a_{13}F_2 + a_{14}R_2, N_0 = a_{12} + a_{13}F_0 + a_{14}R_0, \\ P_1 &= a_{12} + a_{23}F_1 + a_{24}R_1, P_2 = a_{22} + a_{23}F_2 + a_{24}R_2, \\ P_0 &= -a_{23}F_0 + a_{24}R_0, \Delta_0 = c_{13}c_{24} - c_{14}c_{23}, \\ q_1 &= \frac{Z_2}{\Delta_2}, q_2 = -\frac{Z_1}{\Delta_2}, q_3 = q_1 Q_1 + q_2 Q_2, q_4 = q_1 T_1 + q_2 T_2, \\ n_1 &= [(b_{23}Q_0 + b_{24}T_0)M_2 - (b_{13}Q_0 + b_{14}T_0)Z_2] / \Delta_2, \\ n_2 &= [(b_{13}Q_0 + b_{14}T_0)Z_1 - (b_{23}Q_0 + b_{24}T_0)M_1] / \Delta_2, \\ n_3 &= n_1 Q_1 + n_2 Q_2, n_4 = n_1 T_1 + n_2 T_2, \Delta_2 = M_1 Z_2 - M_2 Z_1, \\ Q_1 &= (b_{14}b_{21} - b_{24}b_{11}) / \Delta_{01}, Q_2 = (b_{14}b_{22} - b_{24}b_{12}) / \Delta_{01}, Q_0 = b_{24} / \Delta_{01}, \\ T_1 &= (b_{23}b_{21} - b_{13}b_{21}) / \Delta_{01}, T_2 = (b_{23}b_{12} - b_{13}b_{22}) / \Delta_{01}, T_0 = -b_{23} / \Delta_{01}, \\ M_1 &= d_{12} + d_{13}Q_1 + d_{14}T_1, M_2 = d_{11} + d_{13}Q_2 + d_{14}T_2, M_0 = d_{12} + d_{13}Q_0 + d_{14}T_0, \\ Z_1 &= d_{12} + d_{23}Q_1 + d_{24}T_1, Z_2 = d_{22} + d_{23}Q_2 + d_{24}T_2, \\ Z_0 &= -d_{23}Q_1 + d_{24}T_0, \Delta_0 = b_{13}b_{24} - b_{14}b_{23} \end{aligned}$$

Substituting the expressions  $C_i$  and  $B_i$  from (28) into (18), we obtain

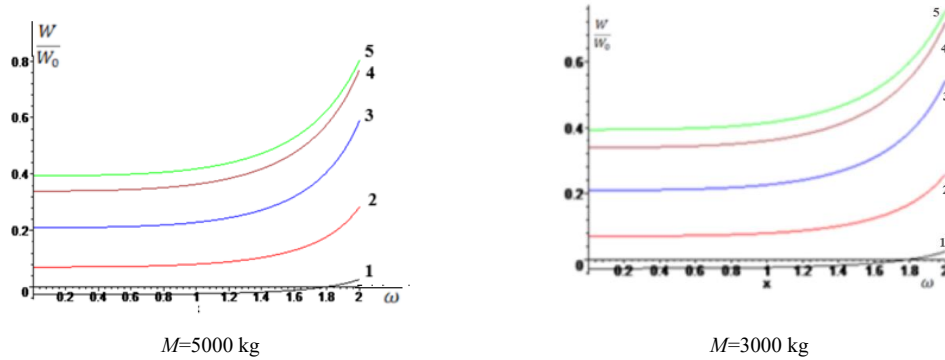
$$W = W_0 \frac{\omega U_0}{\alpha + \omega U_1}$$

where  $\alpha = M/mH$

$$\begin{aligned} U_0 &= a_{12}s_1 + a_{13}s_2 + a_{14}s_3 + a_{11}s_4 + d_{12}n_1 + d_{13}n_2 + d_{14}n_3 + d_{11}n_4 \\ U_1 &= a_{12}r_1 + a_{13}r_2 + a_{14}r_3 + a_{11}r_4 + d_{12}q_1 + d_{13}q_2 + d_{14}q_3 + d_{11}q_4 \end{aligned}$$

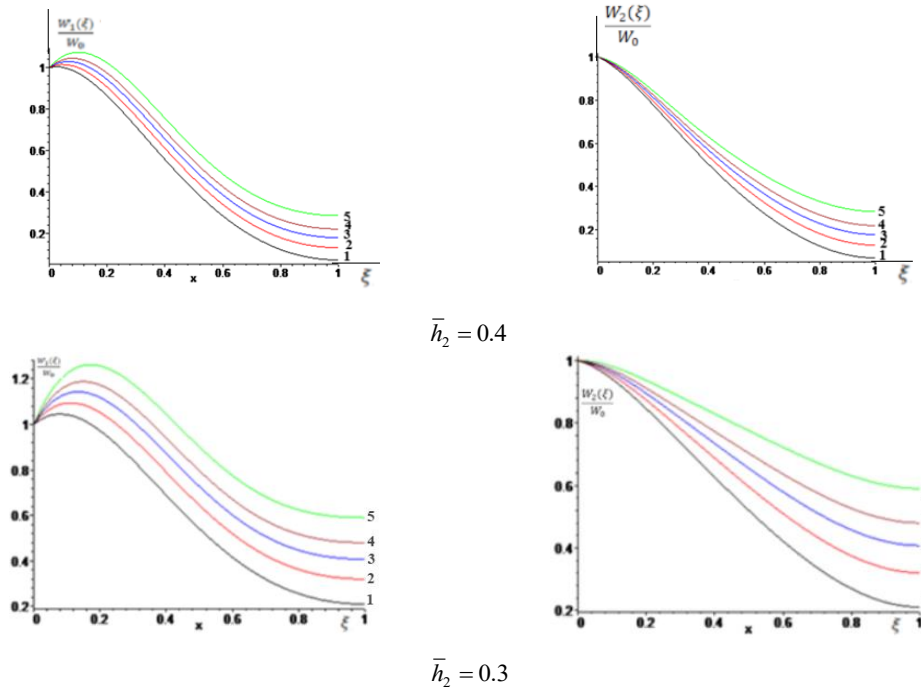
## RESULTS

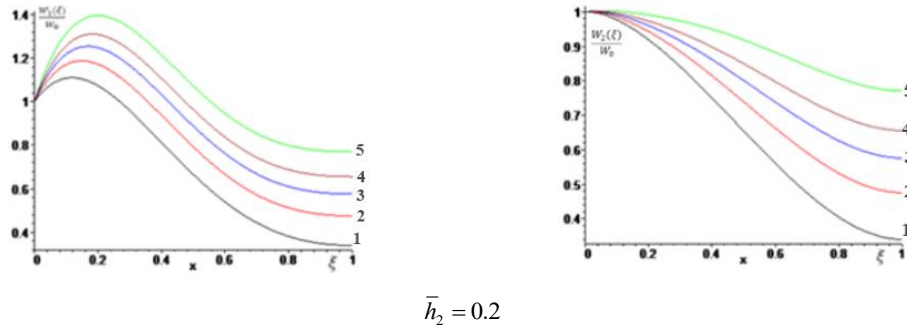
Fig.4 shows the curves of the dependence of the displacements of a rigid beam  $W$  (related to  $W_0$ ) on the dimensionless frequency  $\omega$ . The distribution curves of deflections  $W_1$  and  $W_2$  (related to  $W_0$ ) and longitudinal stresses  $\sigma_i = Ea \frac{d^2 W_i}{2dx^2}$  ( $i=1,2$ ,  $a$  is the length of the sides of the square) along each pile for different values, dimensionless frequency and the depth of the second pile  $h_2$  (related to the length of the pile) in the soil environment are shown in Fig.5 and 6, where in the calculations it is assumed that  $E=4 \cdot 10^4$  MPa,  $k_1=k_2=10^5$  N/m<sup>2</sup>,  $M=5000$  kg,  $a=0.3$  m,  $H=10$  m,  $\bar{h}_1 = 0.8$ .



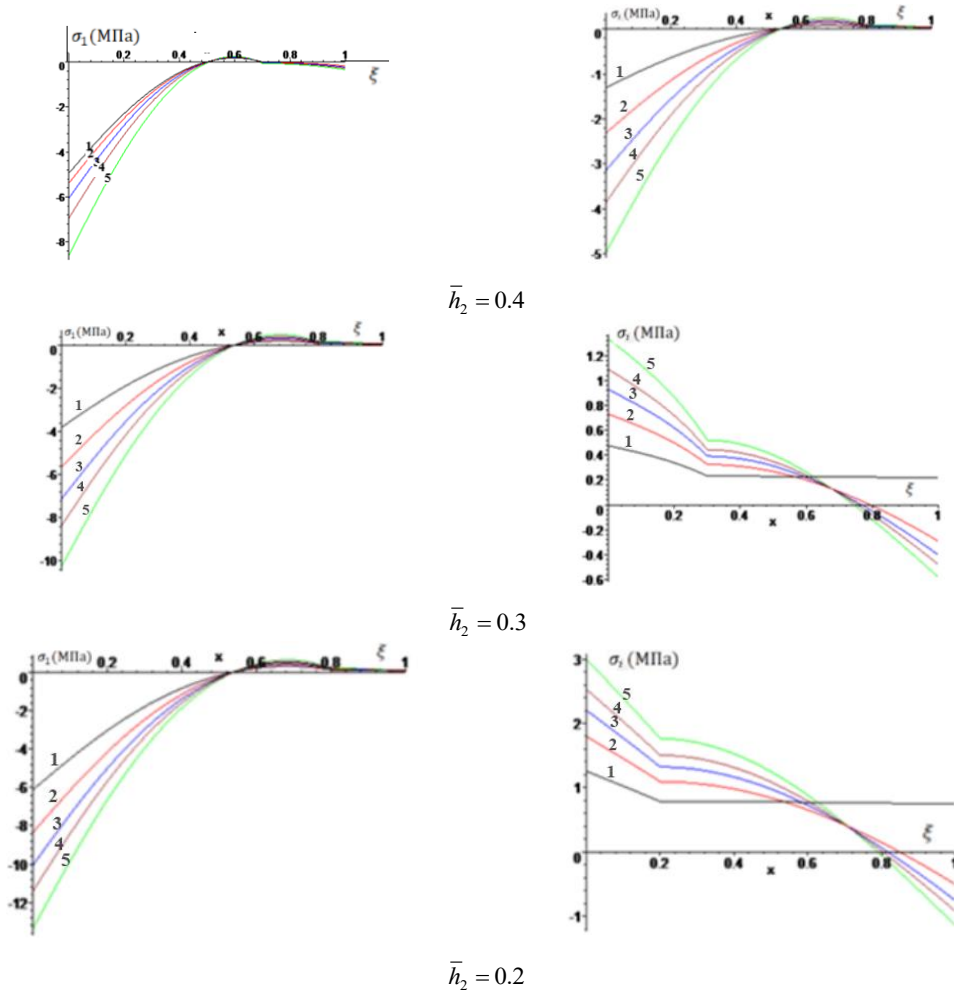
**FIGURE 4.** Dependences of the displacement of a rigid beam on the dimensionless frequency  $\omega$  for two values of its mass  $M$  and the data on the depth  $h_2$  of the second pile (related to  $H$ ):

From the analysis of the curves presented in Fig.6, it follows that when the frequency  $\omega$  changes in the range  $0 < \omega < 1$ , the value of the displacement of the rigid beam remains constant and with a decrease in the depth of the second pile, its amplitude increases. An increase in the beam's mass also leads to an increase in its displacement.





**FIGURE 5.** Distribution curves of deflections (related to  $W_0$ ) of a) the first pile and b) the second pile at  $M=5000$  kg, for different values of the dimensionless frequency  $\omega$  and the embedment depth  $h_2$  of the second pile (related to  $H$ ): 1- $\omega=0.05$ , 2- $\omega=1.6$ , 3- $\omega=1.8$ , 4- $\omega=1.9$ , 5- $\omega=2$



**FIGURE 6.** Stress distribution curves along the length of a) the first pile and b) the second pile at  $M=5000$  kg, for different values of the dimensionless frequency  $\omega$  and the embedment depth  $h_2$  of the second pile (related to  $H$ ): 1- $\omega=0.05$ , 2- $\omega=1.6$ , 3- $\omega=1.8$ , 4- $\omega=1.9$ , 5- $\omega=2$



## CONCLUSION

The nature of the oscillatory process of two piles connected to each other and bearing rigidity depends significantly on the difference in the depth of their installation in the soil environment. If the depth of one pile is maintained and the other is lowered, the pile leads to an increase in the amplitude of vibration of the massive non-deformable beam (slab) rigidly connected to the piles (Fig.4). An analysis of the results of calculations of deflections and stresses shows that the nature of the distribution of deflections along the length of two piles with different depths differs significantly from each other. Moreover, for a pile with a greater depth of embedment, its deflections near the contact with the foundation reach the maximum value, while for another pile the reduction in deflections along the length continuously decreases (Fig.5). From the analysis of the laws of stress distribution along the length of piles (Fig. 6), it follows that the maximum stress values arise in the lower sections of piles, and their greatest values are achieved in piles with a greater depth of embedment in the soil environment.

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