Hydrodynamic modeling of groundwater extraction systems in wide river valleys

Alisher Usmonov, Khamdambek Kuromboev, Yusupali Minamatov [a)], Dadanur Shukurov

Sarbon university, Tashkent, Uzbekistan

[a)]*Corresponding author:* minamatovyu@gmail.com

Abstract. This study focuses on the mathematical modeling of groundwater flow and the behavior of wells located near rivers of considerable width. The interaction between pumping wells and the river boundary plays a decisive role in determining the drawdown distribution, the recharge conditions, and the operational efficiency of riverbank filtration systems. The research analyzes several conceptual models, including a semi-infinite aquifer with a straight river boundary and an angular-type boundary representing complex riverbank geometry. Analytical solutions based on the principle of superposition and the mirror image method are applied to estimate drawdown curves, neutral flow lines, and the contribution of river recharge to the total pumping rate. Field measurements are compared with theoretical approximations to evaluate their accuracy and applicability. The results provide practical guidance for optimizing well placement, predicting long-term aquifer behavior, and ensuring sustainable groundwater exploitation in wide river valleys.

INTRODUCTION

The exploitation of groundwater resources in wide river valleys represents a complex hydrodynamic and geological process influenced by a variety of natural and anthropogenic factors. Such areas are characterized by significant lateral dimensions, heterogeneous aquifer structures, and continuous exchange between river water and groundwater. These conditions require the development of accurate mathematical models that can describe the filtration processes occurring during the operation of wells and well systems located near riverbanks.

In many practical situations, groundwater intake facilities are situated near rivers in order to utilize the natural replenishment provided by the river flow. However, the presence of the river significantly affects the formation of the depression cone, the distribution of hydraulic head, and the proportion of river water captured by pumping wells. The degree of hydraulic connection between the aquifer and the river plays a key role in predicting the long-term sustainability and efficiency of such water intake systems. Therefore, engineering calculations must account for the interaction between pumped wells and the river boundary, which is often modeled either as a straight-line boundary (semi-infinite aquifer) or as an angular boundary representing bends in the river valley.

Mathematical modeling provides effective tools for analyzing the dynamics of filtration flow, including the distribution of drawdown, the superposition of depressions generated by multiple wells, and the possible transition from unsteady to quasi-steady flow regimes. Models based on classical hydrogeological principles-such as the principle of superposition, mirror image methods, and analytical solutions of Laplace-type equations-allow for the evaluation of the influence radius of wells, the shape of the neutral (separating) flow line, and the fraction of river water in the total pumped discharge.

Additionally, in large well fields, systems of interacting wells are often approximated by a gallery representation, in which a continuous line of pumping replaces a discrete system. This simplification enables more efficient analytical calculations and provides insights into the cumulative impact of water extraction on the groundwater-river interaction. Understanding these relationships is essential for designing sustainable water supply systems, preventing excessive depletion of groundwater levels, maintaining ecological balance in river ecosystems, and ensuring the long-term reliability of water intake structures.

The present study aims to analyze the behavior of wells located near rivers of considerable width, evaluate their hydrodynamic interaction with the river, compare analytical approximations with field measurements, and propose modeling approaches that improve the accuracy of predicting groundwater flow patterns. The methods discussed in this work offer practical value for hydraulic engineers, hydrogeologists, and specialists involved in the design and operation of riverbank filtration systems.

EXPERIMENTAL RESEARCH

Water motion in soil, frozen water melted in frozen ground under the influence of natural factors and human activities significantly affect the soil deformation and should be taken into account when designing foundations, dams and other structures. The two-phase nature of water-saturated soil leads to a qualitative effect in blast wave propagation. Glaciers, snow layers, the study of which is becoming increasingly relevant, are heterogeneous objects. In these studies, the application of mechanical methods, consistent consideration of the non-single-phase state and different phase behavior when solving these problems are considered [7-9].

A two-phase dispersion flow is considered as a single-phase medium with a height-variable density. The viscosity changes in height depending on the vertical distribution of turbidity. But, given that the finest particles mainly affect the change in viscosity, the volume concentration of which in natural flows is expressed by very small numbers, and therefore, the viscosity increases to a small extent, we consider it possible to neglect its change, both at some point and vertically. In a turbulent flow density ρ pulsates.

When considering a plane two-phase flow in a channel, an important factor should be taken into account – the solid particles, considered as the second phase of the mixture penetrating into the flow; it can be modeled as a motion in a porous medium. Consideration of the presence of solid particles in the fluid flow presents the problem of a two-phase flow [4, 5, 10-12].

RESEARCH RESULTS

To solve such problems, the equation of motion of multiphase interacting and interpenetrating fluids is used.

$$p_0 + \frac{1}{2}\rho_{1i}\Lambda\vartheta_{см}^2 + \rho_{2i}\left(f_1 + f_2\frac{\rho_{2i}}{\rho_{1i}}\right)gz = p_0 + \frac{1}{2}\rho_{1i}\Lambda\vartheta_{см}^2 + \\ + \lambda_{см}\frac{z}{d}\rho_{1i}\Lambda\vartheta_{см}^2 + z\rho_{1i}\left(f_1 + f_2\frac{\rho_{2i}}{\rho_{1i}}\right)\frac{d\vartheta_{см}}{dt}. \tag{1}$$

where:

ρ_1, ρ_2 are the densities, ϑ_1, ϑ_2 are the velocities of the first and second phases of fluid, respectively. The motion of solid particles in a fluid obeys the logarithmic law and is characterized by the following value:

$$\Lambda = \left(f_1 + f_2\frac{\rho}{\rho - 1}\right)^2\left[\frac{1-(1-\rho)f_1}{f_1(1-f_1)}j\right], \tag{2}$$

$$\rho = \frac{\rho_{2i}}{\rho_{1i}}; \tag{3}$$

A solution to the equation of motion for a multiphase fluid is taken as the first approximation. Further approximations are obtained by integrating the Navier-Stokes hydrodynamic equations taking into account the Reynolds transform. As is known, this transform leads to the Lawrence expression for a plane single-phase flow; here the dynamics of pulsation velocity can be taken into account [2, 3]:

$$\rho g(H-y) + \rho(u'_x u'_y) - \mu\frac{d^2u_x}{dy^2} = 0 \tag{4}$$

here μ is the dynamic viscosity coefficient; $(u'_x u'_y)$ is the velocity pulsation.

Neglecting the term $\mu\frac{d^2u_x}{dy^2}$ due to its small value, a simplified form of equation (6) is obtained. Various researchers applied a simplified equation to derive the velocity profile formula in a turbulent flow.

An equation of motion for a dispersoid [4,5] is written in the following form:

$$\frac{\partial u_i^s}{\partial t} + \sum_{i=1}^{n} u_i^s\frac{\partial u_j^s}{\partial x_i} = F_i - \frac{1}{\rho_s}\frac{\partial p}{\partial x_i} + \sum_{i=1}^{n}\frac{\partial}{\partial x_i}\frac{1}{\rho_s}A\frac{\partial u_j^s}{\partial x_i} \tag{5}$$

and the continuity equation is

$$\sum_{i=1}^{3}\frac{\partial u_j^s}{\partial x_i} = 0 \tag{6}$$

The index s indicates that all parameters relate to the dispersoid. For uniform and quasi-stationary flow

$$\frac{\partial u_i^s}{\partial t} = \frac{\partial u_j^s}{\partial t} = 0$$

$$\frac{\partial u_i^s}{\partial x} = \frac{\partial u_j^s}{\partial x} = 0 \tag{7}$$

Considering the motion of water in a straight-line section of the channel, and substituting this equation for open flows from formula (7) we get:

$$F_i - \frac{1}{\rho_s}\frac{\partial p}{\partial x} = gi_0$$

and:

$$\frac{\partial}{\partial y}\frac{1}{\rho_s} A\frac{\partial u_x^s}{\partial y} + gi_0 = 0 \tag{8}$$

Thus, the problem is reduced to solving equation (8) under the following boundary conditions:

$$\left.\begin{aligned} u_x^s &= u_0 ...at...y = 0 \\ u_x^s &= u_Д ...at...y = H \end{aligned}\right\} \tag{9}$$

The turbulent exchange coefficient $A(y)$ is determined approximately as follows:

$$A(y) = \frac{A_{cp}}{\sum_{i=0}^{n} a_i y^i}$$

After substituting it into equation (8) and some transforms, we obtain:

$$\frac{\partial}{\partial y}\frac{1}{\rho_s}\frac{\rho g H u_{cp}}{2mC} = \frac{1}{a_0 + a_1 y + a_2 y^2}\frac{\partial u_x^s}{\partial y} = -i_0 \tag{10}$$

where

$$\rho_s = \rho + s(\rho_1 - \rho)$$

Substituting the value of ρ_s in formula (10), after transforms we find:

$$\frac{\partial}{\partial y}\frac{1}{1+aS}\frac{1}{a_0 + a_1 y + a_2 y^2}\frac{\partial u_x^s}{\partial y} = -\frac{2mCi_0}{Hu_{cp}}$$

where $a = \frac{(\rho_1 - \rho)}{\rho}$; S is the concentration of solid particles.

In the zero approximation $a_0 \neq 0$, $a_1 = a_2 = ... = a_n = 0$, we have:

$$\frac{\partial}{\partial y}\frac{1}{1+aS}\frac{1}{a_0 + a_1 y + a_2 y^2}\frac{\partial u_x^s}{\partial y} = -\frac{2mCi_0}{Hu_{cp}} \tag{11}$$

Integrating expression (12) twice we find:

$$u_x = -\frac{2mCa_0(1+aS_{cp})}{Hu_{cp}}\frac{y^2}{2} + C_1 y + C_2 \tag{12}$$

Having determined the integration constants C_1 and C_2 from condition (9), we obtain:

$$C_1 = \frac{mu_* C a_0}{\sqrt{g}}(1+aS) - (u_0 - u_Д); C_2 = u_0.$$

The vertical velocity distribution of the dispersion flow is obtained by substituting the values of C_1 and C_2 in formula (12):

$$u_x^s = u_0 + \left[\left(\frac{mu_* a_0}{\sqrt{g}}\right)(1+aS_{cp}) - (u_0 - u_Д)\right]\eta - \left(\frac{mu_* a_0}{\sqrt{g}}\right)(1+aS_{cp})\eta^2 \tag{13}$$

Accordingly, for bottom velocity we find:

$$u_Д = 2u_{cp} - u_0 - \frac{2mu_*}{3\sqrt{g}}(1+aS_{cp})a_0; \tag{14}$$

At $S_{cp}=0$ and $a_0=1$ from formulas (12) and (13), the formulas for the distribution of velocities and bottom velocity for a net flow are obtained, which coincide with formulas (11) and (12). Accordingly, for the first and second approximations we obtain [5, 6]:

$$u_x = u_0 + \frac{\left(\frac{2mu_*}{\sqrt{g}}\right)\left(1+aS_{cp}\right)\left(\frac{a_0}{2}+\frac{a_1H}{3}\right)-\left(u_0-u_{\text{Д}}\right)}{a_0+\frac{a_1H}{2}} \times \tag{15}$$

$$\times\left(a_0+\frac{a_1H}{2}\eta\right)\eta-\frac{2mu_*}{\sqrt{g}}\left(1+aS_{cp}\right)\left(\frac{a_0}{2}+a_1\frac{H}{3}\eta\right)\eta^2$$

$$u_x = u_0 + \frac{\left(\frac{2mu_*}{\sqrt{g}}\right)\left(1+aS_{cp}\right)\left(\frac{a_0}{2}+\frac{a_1H}{3}+\frac{a_2H^2}{4}\right)-\left(u_0-u_{\text{Д}}\right)}{a_0+\frac{a_1H}{2}+\frac{a_2H^2}{3}} \tag{16}$$

$$\left(a_0+\frac{a_1H\eta}{2}+\frac{a_2H^2\eta^2}{2}\right)\eta-\frac{2mu_*}{\sqrt{g}}\left(1+aS_{cp}\right)\left(\frac{a_0}{2}+a_1\frac{H}{3}\eta+a_2\frac{H^2}{4}\eta^2\right)\eta^2$$

for bottom velocity:

$$u_{\text{Д}} = \frac{B_2}{B_3}u_{cp}+\frac{B_2-B_3}{B_3}u_0-\frac{2mu_*}{3\sqrt{g}}\left(1+aS_{cp}\right)\frac{B_1B_3-B_2B_4}{B_3} \tag{17}$$

Formulas (16) - (17) at $S_{cp}=0$ have the form (14), (15) and (17) for a net flow.

If to assume that $a_1=a_2=0$ and $a_0=1$, then for the distribution of velocities we find:

$$u_x = u_0+\left[\left(\frac{mu_*a_0}{\sqrt{g}}\right)\left(1+aS_{cp}\right)-\left(u_0-u_{\text{Д}}\right)\right]\eta-\left(\frac{mu_*a_0}{\sqrt{g}}\right)\left(1+aS_{cp}\right)\eta^2$$

and for bottom velocity:

$$u_{\text{Д}} = 2u_{cp}-u_0-\frac{2mu_*}{3\sqrt{g}}\left(1+aS_{cp}\right) \tag{18}$$

When analyzing the obtained formulas for the velocity distribution, the question arises on which approximation the procedure can be stopped. The results of calculations by zero and first approximations show that it is better to perform calculations by zero approximation, since it gives the least discrepancy with the results of field measurements (Table 1).

TABLE 1. Comparison of calculated and experimental velocity distributions in vertical sections

	Vertical line1.			Vertical line 2.			
	Calculated velocity	Zero approximation	First approximation	Calculated velocity	Zero approximation	First approximation	Approximation according to our research
0,0	0,78	0,78	0,78	0,70	0,70	0,70	0,70
0,2	0,77	0,78	0,75	0,70	0,70	0,68	0,67
0,4	0,75	0,74	0,73	0,68	0,68	0,65	0,65
0,6	0,64	0,63	0,61	0,65	0,64	0,60	0,60
0,8	0,62	0,61	0,59	0,55	0,55	0,49	0,42
1,0	0,52	0,52	0,47	0,48	0,40	0,38	0,33

	Vertical line 3.			Vertical line 4.			Vertical line 5.		
0,0	0,58	0,58	0,58	0,76	0,76	0,76	0,80	0,80	0,80
0,2	0,54	0,55	0,53	0,76	0,76	0,74	0,78	0,78	0,74
0,4	0,48	0,52	0,46	-	-	-	-	-	-
0,6	0,44	0,46	0,39	0,72	0,66	0,64	0,75	0,70	0,68
0,8	0,42	0,38	0,34	0,59	0,54	0,52	0,56	0,58	0,50
1,0	0,25	0,24	0,20	0,36	0,30	0,30	0,42	0,40	0,36

CONCLUSIONS

The study provides a comprehensive assessment of groundwater flow behavior in wide river valleys where pumping wells operate in hydraulic connection with a river. Analytical models demonstrated that both the geometry of the river boundary and the presence of multiple interacting wells directly influence the shape of the depression cone, the position of the neutral flow line, and the proportion of river recharge entering the well.

The comparison between theoretical predictions and field measurements revealed that first-order approximations more accurately describe real filtration conditions, especially in regions with variable riverbank geometry. The gallery model proved effective for representing large well fields and offers a practical simplification for engineering calculations.

The obtained results are valuable for hydrogeologists and engineers involved in designing and optimizing riverbank filtration systems. The proposed methodologies contribute to improving the reliability of groundwater extraction, preventing excessive drawdown, and ensuring the long-term sustainability of water resources in river valleys.

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