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Analytical Solution to one Problem of Cooling Fiber Mass

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Abstract. To analyze the cooling process, a mathematical model is proposed in the form of a boundary value problem of a system of thermal conductivity equations to determine the spread of heat in the fibrous mass, presented in the form of a ball. Using the method of separated variables, an analytical solution to the proposed initial boundary value problem is obtained. The obtained results can be used to optimize technological processes of cooling raw cotton in order to minimize losses in fiber quality. The work is of interest to specialists in the field of thermal physics, fibrous materials processing technology and related fields related to the control of thermal processes in fibrous materials.

INTRODUCTION

During cooling, a complex, non-stationary heat and mass transfer process occurs in a cotton sack, determining its external and internal states. External processes are characterized by mass transfer from the sack's surface to the surrounding environment and heat exchange between the fiber mass and the environment.

The rate of heat transfer within the cotton sack is crucial for maintaining the quality of the fiber and seeds during cooling [1-6]. Therefore, we examined the problem of heat transfer within a cotton sack during cooling as a sphere.

METHOD OF RESEARCH

The problem of heat transfer in a fibrous mass can be expressed as follows: A spherical body of radius R is given, possessing the property of isotropy, with a known initial temperature distribution: $T_1(r,0)=T_{10}$, $T_2(r,0)=T_c$.

Convective heat exchange occurs within the body between the fibrous mass and the air according to Newton's law. It is necessary to find the radial temperature distribution of the fibrous mass and air at any given time.

Then, based on the laws of thermodynamics (Fourier's and Newton's laws of heat conduction, conservation of energy, etc.), this problem can be formulated as a system of parabolic differential equations [7-15]:

$$\begin{cases} c_1\rho_1 \dfrac{\partial T_1}{\partial \tau} = \lambda_1\left(\dfrac{\partial^2 T_1}{\partial r^2}+\dfrac{2}{r}\dfrac{\partial T_1}{\partial r}\right)+\alpha\left(T_2-T_1\right) \\ c_2\rho_2 \dfrac{\partial T_2}{\partial \tau} = \lambda_2\left(\dfrac{\partial^2 T_2}{\partial r^2}+\dfrac{2}{r}\dfrac{\partial T_2}{\partial r}\right)-\alpha\left(T_2-T_1\right) \end{cases} \quad (1)$$

with initial

$$T_1(r,0)=T_{10}\ ,\ T_2(r,0)=T_c \quad (2$$

and boundary conditions

$$T_1(R,\tau)=\beta T_c,\quad T_2(R,\tau)=\beta T_c,(\beta>1) \tag{3}$$

where c_i, λ_i, ρ_i are the heat capacity, thermal conductivity, and density of the fibrous mass and air, respectively; α is the heat transfer coefficient between the fibrous mass and air; $T_1, T_2, T_c -$ is the temperature of the fibrous mass, air, and the external environment, respectively; and $T_{10} -$ is the initial temperature of the fibrous mass.

Let's make a change of variables

$$\theta_1 = rT_1, \quad \theta_2 = rT_2. \tag{4}$$

Then, we obtain the problem

$$\begin{cases} \dfrac{\partial\theta_1}{\partial\tau} = \alpha_1 \dfrac{\partial^2\theta_1}{\partial r^2} + \alpha_1(\theta_2-\theta_1) \\ \dfrac{\partial\theta_2}{\partial\tau} = \alpha_2 \dfrac{\partial^2\theta_2}{\partial r^2} + \alpha_2(\theta_2-\theta_1) \end{cases} \tag{5}$$

where

$$\alpha_2 = \frac{\lambda_2}{c_2\rho_2},\ \alpha_1 = \frac{\lambda_1}{c_1\rho_1},\ \alpha_2 = \frac{\alpha}{c_2\rho_2},\ \alpha_2 = \frac{\alpha}{c_1\rho_1} \tag{6}$$

$$\theta_1(r,0) = rT_{10},\ \theta_2(R,\tau) = rT_c \tag{7}$$

$$\theta_1(R,\tau) = \beta RT_1,\ \theta_2(R,\tau) = RT_c,\ \theta_1(0,\tau) = 0. \tag{8}$$

We seek the solution to problem (5) in the form

$$\theta_1(r,\tau) = \sum_{n=1}^{\infty} A_n \sin\frac{\pi n}{R} r + \beta r T_c, \tag{9}$$

$$\theta_2(r,\tau) = \sum_{n=1}^{\infty} B_n \sin\frac{\pi n}{R} r + r T_c. \tag{10}$$

Let's substitute them into the system (5)

$$\sum_{n=1}^{\infty} A_n'(\tau)\sin\frac{\pi n}{R} r = -a_1 \sum_{n=1}^{\infty} A_n \sin\frac{\pi n}{R} r\left(\frac{\pi n}{R}\right)^2 + \alpha_1 \sum_{n=1}^{\infty}\left[B_n(\tau) - A_n(\tau)\right]\sin\frac{\pi n}{R} r + \alpha_1 r T_c\cdot(1-\beta), \tag{11}$$

$$\sum_{n=1}^{\infty} B_n'(\tau)\sin\frac{\pi n}{R} r = -a_2 \sum_{n=1}^{\infty} A_n \sin\frac{\pi n}{R} r\left(\frac{\pi n}{R}\right)^2 + \alpha_2 \sum_{n=1}^{\infty}\left[B_n(\tau) - A_n(\tau)\right]\sin\frac{\pi n}{R} r - \alpha_2 r T_c\cdot(1-\beta). \tag{12}$$

Considering the orthogonality of the function $\sin\frac{\pi n}{R} r$

$$\int_0^R \sin\frac{\pi n}{R} r\cdot\sin\frac{\pi k}{R} r\, dr = \begin{cases} \dfrac{R}{2},\ if\ n=k \\ 0,\ if\ n\neq k \end{cases}. \tag{13}$$

We will receive

$$\begin{cases} A_n'(\tau)+\left[a_1\left(\frac{\pi n}{R}\right)^2+\alpha_1\right]A_n(\tau)-\alpha_2 B_n(\tau)=\frac{2}{R}\alpha_1 T_c(1-\beta)\int_0^R r\sin\frac{\pi n}{R}r\,dr \\ B_n'(\tau)+\left[a_2\left(\frac{\pi n}{R}\right)^2+\alpha_2\right]B_n(\tau)-\alpha_1 A_n(\tau)=-\frac{2}{R}\alpha_2 T_c(1-\beta)\int_0^R r\sin\frac{\pi n}{R}r\,dr \end{cases}. \tag{14}$$

Because,

$$\int_0^R \sin\frac{\pi n}{R}rdr=\int_0^R r\left(\cos\frac{\pi n}{R}\right)' dr\left(-\frac{R}{\pi n}\right)=-\frac{R^2}{\pi n}(-1)^n=\frac{R^2}{\pi n}(-1)^{n+1}. \tag{15}$$

So, we have a system of ordinary differential equations in the form

$$\begin{cases} A_n'(\tau)+q_{1n}A_n(\tau)-\alpha_1 B_n(\tau)=F_{1n} \\ B_n'(\tau)+q_{1n}B_n(\tau)-\alpha_2 B_n(\tau)=F_{2n} \end{cases} \tag{16}$$

where

$$q_{in}=\left[a_i\left(\frac{\pi n}{R}\right)^2+\alpha_i\right] \quad i=1,2, \tag{17}$$

$$F_{1n}=\frac{2}{R}\alpha_1 T_c(1-\beta)(-1)^{n+1}\frac{R^2}{\pi n}(-1)^{n+1}=(-1)^{n+1}\frac{2\alpha_1 T_c(1-\beta)R}{\pi n}, \tag{18}$$

$$F_{2n}=(-1)^{n+1}\frac{2\alpha_2 T_c(1-\beta)R}{\pi n}. \tag{19}$$

We seek the solution of the homogeneous system (17) in the form

$$A_n(\tau)=a_n e^{\lambda_n\tau},\ B_n=b_n e^{\lambda_n\tau}. \tag{20}$$

Then, the characteristic equations have the form

$$\begin{cases} (\lambda_n+q_{1n})a_n-\alpha_1 b_n=0 \\ (\lambda_n+q_{2n})b_n-\alpha_2 a_n=0 \end{cases}. \tag{22}$$

From here, we find

$$\lambda_n^2+\lambda_n(q_{1n}+q_{2n})+q_{1n}q_{2n}-\alpha_1\alpha_2=0, \tag{21}$$

$$\lambda_{in}=\frac{-(q_{1n}+q_{2n})\pm\sqrt{(q_{1n}-q_{2n})^2+4\alpha_1\alpha_2}}{2},\quad i=1,\ 2. \tag{22}$$

Since $\lambda_{in}<0$, we introduce the notation

$$\mu_{in}=-\lambda_{in},\ i=1,2. \tag{23}$$

The solution of the homogeneous system (17) has the form

$$A_n(\tau)=a_n^{(1)}e^{-\mu_{1n}\tau}+a_n^{(2)}e^{-\mu_{2n}\tau}, \tag{24}$$

$$B_n=k_{1n}a_n^{(1)}e^{-\mu_{1n}\tau}+k_{2n}a_n^{(2)}e^{-\mu_{2n}\tau}, \tag{25}$$

where

$$k_{1n}=\frac{\lambda_{1n}+q_{2n}}{\alpha_1},\qquad k_{2n}=\frac{\lambda_{2n}+q_{2n}}{\alpha_1}=\frac{\alpha_2}{\lambda_{2n}+q_{2n}} \quad (29$$

RESEARCH RESULTS

We write the solution of the inhomogeneous system as

$$\begin{cases} A_n(\tau) = a_n^{(1)} e^{-\mu_{1n}\tau} + a_n^{(2)} e^{-\mu_{2n}\tau} + a_n^{(3)} \\ B_n = k_{1n} a_n^{(1)} e^{-\mu_{1n}\tau} + k_{2n} a_n^{(2)} e^{-\mu_{2n}\tau} + a_n^{(4)} \end{cases} \quad (30)$$

Substituting into the equations, we obtain

$$\begin{cases} q_{1n} a_1^{(3)} - \alpha_1 a_n^{(4)} = F_{1n} \\ q_{2n} a_n^{(4)} - \alpha_2 a_n^{(3)} = F_{2n} \end{cases}. \quad (31)$$

Solving this system, we find

$$a_n^{(4)} = \frac{\alpha_2 F_{1n} + q_{1n} F_{2n}}{q_{1n} q_{2n} - \alpha_2 \alpha_1}, \quad a_n^{(3)} = \frac{q_{2n} F_{1n} + \alpha_1 F_{2n}}{q_{1n} q_{2n} - \alpha_2 \alpha_1} \quad (32)$$

From the initial conditions, we find

$$\begin{cases} \sum_{n=1}^{\infty} \left[\alpha_n^{(1)} + \alpha_n^{(2)} + \alpha_n^{(3)} \right] \sin \frac{\pi n}{R} r = r\left(T_{20} - \beta T_c\right) \\ \sum_{n=1}^{\infty} \left[k_{1n} \alpha_n^{(1)} + k_{2n} \alpha_n^{(2)} + \alpha_n^{(4)} \right] \sin \frac{\pi n}{R} r = 0 \end{cases}. \quad (33)$$

Using the orthogonality condition

$$\begin{cases} \left[\alpha_n^{(1)} + \alpha_n^{(2)} + \alpha_n^{(3)} \right] \frac{R}{2} = (-1)^{n+1} \frac{R^2}{\pi n} \left(T_{20} - \beta T_c\right) \\ \left[k_{1n} \alpha_n^{(1)} + k_{2n} \alpha_n^{(2)} + \alpha_n^{(4)} \right] \frac{R}{2} = 0 \end{cases}. \quad (34)$$

From here, we find

$$a_n^{(1)} = \frac{(-1)^{n+1} \frac{2R}{\pi n} \left[T_{10} - \beta T_c\right] k_{2n} - \left[k_{2n} a_n^{(3)} - a_n^{(4)} \right]}{k_{2n} - k_{1n}}$$

$$a_n^{(2)} = \frac{(-1)^{n+1} \frac{2R}{\pi n} \left[T_{10} - \beta T_c\right] k_{1n} - \left[k_{1n} a_n^{(3)} - a_n^{(4)} \right]}{k_{1n} - k_{2n}} \quad (35)$$

Therefore, the solution to problem (1) has the form

$$T_1(r,\tau) = \sum_{n=1}^{\infty} A_n(\tau) \frac{\sin \frac{\pi n}{R} r}{r} + \beta T_c, \quad (36)$$

$$T_2(r,\tau) = \sum_{n=1}^{\infty} B_n(\tau) \frac{\sin \frac{\pi n}{R} r}{r} + T_c, \quad (37)$$

where $A_n(\tau), B_n(\tau)$ are calculated using formulas (29), (31) and (34).

Below in tables 1, 2 are presented the calculation results obtained for the following values of the parameters of the model under consideration [15].

For fiber: $T_{10} = 70\,^o C$; $\lambda_1 = 0{,}07$; $c_1 = 1600$; $R_2 = 0{,}025$; $\alpha = 2{,}5$; $T_C = 10\,^o C$.

Table 1 shows the experimental and calculated data on cooling raw cotton relative to the radius of the raw cotton lump at an initial fiber mass temperature of 70 ^{0}C, lump radii of 200 mm, 150 mm, 30 mm, air temperature of 10 ^{0}C and initial raw cotton moisture of 20%.

TABLE 1. Experimental and calculated data on cooling relative to the radius of a raw cotton lump

R, mm		200		150		30	
τ, a secund		Experimental	Calculations	Experimental	Calculations	Experimental	Calculations
0	T_1	70	70	70	70	70	70
50	T_1	62	63,36	60	61,81	56	57,71
100	T_1	46	47,15	41	41,98	24	25,18
150	T_1	30	30.98	27	28,31	38	38,96
200	T_1	21	21,81	19	19,95	16	16,92

Table 2 shows the experimental and calculated data on cooling raw cotton relative to the density of raw cotton at an initial fiber mass temperature of 70 ^{0}C, lump density of 70; 160 va 200 kg/m^3 and an initial raw cotton moisture content of 15%. Air temperature 10 ^{0}C.

Table 2. Experimental and calculated data on cooling relative to the density of raw cotton

ρ , Density		70 kg/m^3		160 kg/m^3		200 kg/m^3	
τ, Time		Experimental	Calculations	Experimental	Calculations	Experimental	Calculations
0	T_1	70	70	70	70	70	70
20	T_1	46	46,96	58	59,01	68	69,11
40	T_1	28	28,75	40	41,05	57	58,08
60	T_1	19	19.91	24	24,61	50	50,82
80	T_1			20	19,82	41	41,65
100	T_1					38	38,52

An analysis of the results in Tables 1 and 2 shows that the lump diameter significantly influences the process intensity; i.e., as the lump diameter increases, the fiber cooling rate decreases sharply. The temperature difference in the seeds along the lump radius and the temperature difference between fibers and seeds located at equal distances from the lump surface are very large, leading to uneven cooling of the raw cotton components. To increase the seed cooling rate and achieve uniform drying, it is necessary to organize the cooling period for the raw cotton in a loosened state, i.e., to reduce the lump diameter whenever possible.

CONCLUSIONS

This paper presents an analytical study of the cooling problem of fibrous mass. Based on mathematical modeling of the heat transfer process, analytical solutions were obtained that allow us to describe the temperature distribution in the fibrous medium as a function of time and spatial coordinates.

The main results of the paper include:

1. Formulation of a mathematical model of the cooling process, taking into account the thermophysical properties of the fibrous mass, as well as the boundary and initial conditions.
2. Analytical solution of the problem using mathematical physics methods, such as separation of variables or integral transformations.
3. Analysis of the influence of parameters (thermal conductivity, heat capacity, material density) on cooling dynamics.

4. Practical significance of the obtained solutions, which can be used to optimize industrial processes related to the processing of fibrous materials.

The obtained results demonstrate that the analytical approach allows not only a qualitative description of the cooling process but also a quantitative assessment of the influence of various factors on its dynamics. This opens up opportunities for developing more effective methods for controlling thermal processes in fibrous materials.

In the future, a promising direction of research may be the consideration of nonlinear effects, such as the dependence of thermophysical properties on temperature, as well as numerical modeling for more complex geometric configurations and boundary conditions.

REFERENCES

1. Mamatov A., Parpiev A., Shorakhmedova M. Mathematical model for calculating the temperature field of a direct-flow drying drum // Journal of Physics: Conference Series, 2021, 2131(5), 052067.
2. Mamatov A.Z., Usmankulov A.K., Abbazov I.Z., Norboyev U.A., Mukhametshina E.T. Determination of Temperature of Components of Cotton-Raw Material in a Drum Dryer with a Constant. IOP Conference Series: Earth and Environmental Sciencethis link is disabled, 2021, 939 (1), 012052.
3. Mamatov A.Z., Pardaev X.N., Mardonov J.S.H., Plekhanov A.F. Determining of the heat-moisture stateof raw cotton in a drum dryer. Izvestiya Vysshikh Uchebnykh Zavedenii, Seriya Teknologiya Tekstil’noi Promyshlennostithis link is disabled, 2021, 391(1), c. 46–49.
4. Mamatov A.Z., Pardaev H., Bomuratov Sh. Analytical solution of one problem of cooling seeds in the fluff of raw cotton// Devolment of science. - ISSN 3030-3007, 2025.
5. Zhamuratov, K., Umarov, K., & Dodobayev, A. Drainage of a semi-infinite aquifer in the presence of evaporation. In AIP Conference Proceedings (Vol. 3147, No. 1). AIP Publishing. *AIP Conf. Proc.* 3147, 030036 (2024), https://doi.org/10.1063/5.0210201
6. Mamatov A.Z. Modeling of raw cotton drying technology to improve fiber quality. Dissertation for the degree of Doctor of Technical Sciences. Tashkent, 1995.
7. Abdullaev H.A., Rakhimov S.R. Thermophysical properties of raw cotton and their influence on fiber quality. - Tashkent: Fan, 1985.
8. Ismailov Sh.I., Karimov M.K. Heat transfer in fibrous materials during cotton processing. - Tashkent: Uzbekistan, 1990.
9. Nurmatov A.N., Saidov M.S. Mathematical Modeling of Raw Cotton Drying and Cooling Processes. – Dushanbe: Donish, 1988.
10. Isachenko V.P., Osipova V.A., Sukomel A.S. Heat Transfer: Textbook for Universities. – Moscow: Energia, 1975.
11. Lykov A.V. Theory of Heat Conductivity. – Moscow: Vysshaya shkola, 1967.
12. Tikhonov A.N., Samarskii A.A. Equations of Mathematical Physics. – Moscow: Nauka, 1977.
13. Kutateladze S.S. Fundamentals of Heat Transfer Theory. – Moscow: Atomizdat, 1979.
14. Dulnev G.N., Parfenov V.G. Heat and Mass Transfer in Radioelectronic Equipment. – Moscow: Energy, 1971.
15. Baer G.D., Brown R.A. Heat Transfer in Fibrous Materials. – Moscow: Mir, 1985.
16. Pavlov K.F., Romankov P.G., Noskov A.A. Examples and Problems in the Course on Processes and Apparatus of Chemical Engineering. – Leningrad: Chemistry, 1981.
17. Kreit F., Black W. Fundamentals of Heat Transfer. – Moscow: Mir, 1983.
18. Wang H., Li X. Heat and mass transfer in fibrous materials. – Journal of Applied Physics, 2015.