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Development of an Efficient Method for the Allocation of Motor Vehicles to Routes within the Capabilities of Shipment (Reception) Points

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Abstract. The article discusses the shortcomings of solving the problem of allocating automobile transport vehicles to routes using integer linear programming and proposes a combinatorial method that allows determining discrete solutions for efficient route allocation. In identifying discrete solutions for efficient route allocation, a mathematical model and corresponding algorithms have been developed to ensure that all types of automobile transport vehicles operate within the loading and unloading capacities of the routes.

INTRODUCTION

Modern transport and logistics industries require the extensive application of optimization methods in production processes. Such optimization methods include differential calculus techniques, numerical methods, constrained and unconstrained optimization approaches, as well as variant selection techniques.

Among the most widely used modern approaches is the **linear programming method**, which involves determining the extreme values of a multivariable linear function subject to a system of linear constraints [3,7,9]. In **nonlinear programming**, both the objective function and the constraints may be nonlinear. A special case within linear and nonlinear programming is when the optimal solution must consist of integer values; such problems are classified as **integer programming** tasks [4,5,6].

The distribution problem can also be solved using integer linear programming models; however, when applying linear programming models to the allocation of motor vehicles to routes, several limitations and inconveniences arise [1,10]:

1. In organizing planned freight transportation, it is necessary to determine the rational number of loading and unloading mechanisms required to serve the vehicles assigned to each route.
2. In solving the vehicle-to-route allocation problem, fractional values often need to be rounded to integers, which reduces the degree of optimality of the obtained solution.
3. When determining the solution, the availability of loading and unloading mechanisms operating along each transport route during the daily working period is not considered. Since the number of mechanisms in operation on a given day is always an integer, a certain number of vehicles must be allocated to ensure their full utilization throughout the day.

Such requirements cannot be fully considered within the previously described models. Therefore, it becomes necessary to develop **combinatorial models** for solving the vehicle-to-route allocation problem and to design **discrete solution methods** for these models.

The elements of the discrete set represent the number of vehicles (MVs) that can be assigned to each route. In this case, the number of vehicles that can be allocated to a specific route is determined based on the **loading or service capacities** at the shipment points and their ability to dispatch vehicles within a given time unit. In general, solving this problem involves determining the number of vehicles assigned to each route as **discrete elements** of the overall set [2,8].

MAIN TEXT

When distributing vehicles of **different load-carrying capacities** among routes, it is necessary to determine, for each route, both the number of vehicles assigned for freight transportation according to their type and the corresponding **transportation volumes** to be performed on that route. In formulating this problem, the following **main properties and constraints** should be taken into account [1].

Firstly, the solution to the problem of allocating motor vehicles involved in transportation to the respective inbound or outbound routes determines the **degree to which the transportation demand of consumer destinations is satisfied**. In other words, assigning a larger portion of the available motor vehicles to **shorter routes** increases the total freight volume transported, while allocating more vehicles to **longer-distance routes** tends to decrease the total delivered volume. If the number of vehicles available for transportation is limited, achieving the required freight volume necessitates allocating them mainly to **shorter routes**. Conversely, when a greater number of vehicles are available but the transportation demand is relatively smaller, a larger share of these vehicles should be assigned to **longer routes**.

Secondly, automobile transport vehicles with different load-carrying capacities provide different levels of transportation cost efficiency when operating on routes of various lengths. Therefore, among the many possible variants of vehicle-to-route allocation, it is necessary to determine an optimal option that ensures the fulfillment of consumer demand for freight volume, enables efficient utilization of shipment and reception capacities at each route, and minimizes total transportation costs through the rational distribution of automobile transport vehicles among routes.

Thirdly, the number of automobile transport vehicles assigned to each route must ensure the effective utilization of the available loading and unloading mechanisms at the corresponding shipment or reception points over time. For each *j*-th route, the discrete set of available loading and unloading mechanisms is defined as ∑j = {1, 2, …, σj}. The discrete nature of the distribution problem lies in the fact that the number of automobile transport vehicles assigned to each *j*-th route must be determined based on the criterion of fully utilizing each loading (or unloading) mechanism during the working period, and the total number of planned vehicles must be equal to γi. Therefore, the feasible solutions to the vehicle-to-route allocation problem must consist of elements that belong to a certain discrete set.

In the problem formulation, the number of Automobile Transport Vehicles assigned to each *j*-transport route ($X_{j\sigma_j}^{i}$) must ensure the efficient utilization of each *i*-type of Automobile Transport Vehicle and provide rational loading options for a given number of loading (or unloading) mechanisms σ_j. The objective is to make efficient use of the available Automobile Transport Vehicles $A_{\text{э}}^{i}$), satisfy the consumer's transportation demand with minimum total costs, and ensure the effective utilization of loading (or unloading) mechanisms during the working period.

For the full utilization of the Automobile Transport Vehicles operated during the day, a total of $A_{\text{э}}^{i}$ vehicles must be completely allocated across all routes, meaning they should be distributed in the form of the corresponding sum of $X_{j\sigma_j}^{i}$ vehicles. Therefore, we introduce the concept of a set of vehicles *(m_g)* that meets the requirement for the full utilization of Automobile Transport Vehicles within the demand scope. The set *(m_g)* of Automobile Transport Vehicles of type *i* within the demand scope is defined as the set that satisfies the following condition:

$$\sum_{j\in J} X_{j\sigma_j}^{i} = A_{\text{э}}^{i}, \mathrm{x}_{j\sigma_j} \in G_{j\sigma_{j(M)}}, i \in I \tag{1}$$

Here, $G_{j\sigma_{j(i)}} - X_{j\sigma_j}^{i}$ denote elements of the set of feasible aggregations (i.e., the set of admissible assemblies of vehicles).

$A_{\text{э}}^{i}$ − The number of Automobile Transport Vehicles of type *i*.

The solution of the formulated problem is to determine such a combination of the aggregations of Automobile Transport Vehicles within the demand scope, $X_{j\sigma_j}^{i}$, for all i∊I, that the following condition is satisfied.

$$Q \overset{\Sigma}{\sum_{i\in I}} \sum_{j\in J} X_{j\sigma_j}^{img} \cdot Q_{ij} \le Q_{j\sigma_j}^{img}{}^{mg}{}_{j\sigma_{j_{max}}}{}_{min} \tag{2}$$

Here, $X_{j\sigma_j}^{img}$ − lements of the aggregation within the demand scope.

$G_{j\sigma_j}^{mg}$ − Elements of the aggregation within the demand scope.

Q_{ij} − The daily productivity, in tons, of type *i* Automobile Transport Vehicles on the *j*-transport route.

Q_{min} , Q_{max} - The minimum and maximum levels of the consumer destination's demand for the volume of cargo transportation.

In the above developments, the elements $X^i_{j\sigma_j}$ of the aggregations within the demand scope and their combinations were shown to be formed according to conditions (1) and (2). These aggregations $X^i_{j\sigma_j}$ are aimed at ensuring the full utilization of the Automobile Transport Vehicles of each type *i* that are in operation. The formation of these aggregations for all sets *iϵI* end *iϵJ* is carried out in accordance with conditions (2), which ensure that the volumes of cargo to be delivered to (or shipped from) each consumer destination remain within the specified boundary values. However, one important limitation arising from real transportation parameters has not been taken into account—namely, the total number of Automobile Transport Vehicles of all types *iϵJ* allocated to each *j*-route must not exceed the available loading capacity at that route. Therefore, the solution to the distribution problem must be verified using the following additional conditions, and, if necessary, reformulated accordingly.

The total number of Automobile Transport Vehicles (ATVs) of all types allocated to each transportation route must be verified for consistency with the route's available cargo dispatch (or reception) capacity.

If the total number of vehicles exceeds the route's loading (or unloading) capacity, then appropriate adjustments must be made within the obtained combined aggregations so as to eliminate this inconsistency.

After implementing the necessary adjustments, the newly restructured aggregation must again be checked for compliance with the above condition.

During the verification and reformation processes described above, it is first necessary to substantiate a mathematical model of the condition stating that *the total number of ATVs of all types assigned to a route must remain within the limits of that route's cargo dispatch (or reception) capacity*.

The capacity of route j to perform loading (or unloading) operations depends on the number of mechanisms (machines) in service, denoted σj let σ_j^{max} be the maximum possible value of σj. Thus $\sigma_j \in \{1,2,\ldots\sigma_j\ldots\sigma_j^{max}\{\}\}$ When σ_j=1, a single mechanism is capable of servicing γi Automobile Transport Vehicles of type i (i.e., the number of vehicles that one mechanism can load or unload during the working period is γi). Suppose the numbers of Automobile Transport Vehicles assigned to route j for all types i∈ {1,2,…,I} are known, i.e. Then the number of mechanisms required to serve the vehicles of each type on route jjj can be determined as follows. $X^{i=1}_{j\sigma_j}, X^{i=2}_{j\sigma_j}, \ldots X^{i=I}_{j\sigma_j}$ In this case, the number of loading (or unloading) mechanisms required to serve the Automobile Transport Vehicles of various load-carrying capacities assigned to route jjj is determined as follows:

$\sigma^{i=1}_{j\sigma_j} = X^{i=1}_{j\sigma_j}/\gamma_{i=1}; \sigma^{i=2}_{j\sigma_j} = X^{i=2}_{j\sigma_j}/\gamma_{i=2}$ The number of mechanisms that must serve the loading of vehicles with different carrying capacities distributed along the route is determined as follows:

$$\sigma_j^{i=1} + \sigma_j^{i=2} + \ldots + \sigma_j^{i} \le \sigma_j^{max} \text{ that is,}$$

$$\frac{X^{i=1}_{j\sigma_j}}{\gamma_{i=1}} + \frac{X^{i=2}_{j\sigma_j}}{\gamma_{i=2}} + \cdots + \frac{X^{i=I}_{j\sigma_j}}{\gamma_{i=I}} \le \sigma_j^{max} \;\; or \sum_{i\in I_j} \frac{X^{i}_{j\sigma_j}}{\gamma_i} \le \sigma_j^{max} \tag{3}$$

Here, I_j is the set of vehicle types allocated to route j in the solution of the problem.

In the formed combined aggregations, the verification of compliance with condition (2) is carried out as follows:

Here is the English translation of your text in a formal, scientific style suitable for a Scopus journal, including mathematical context: Based on certain procedures, the aggregation combination within the demand scope $\left\{H(X^1_{j\sigma_j}\ldots H(X^I_{j\sigma_j})\right\}$ is formed, and this combination satisfies condition (1). In this combination, if the aggregations are formed in **"forward"** order, the values of the elements $X^i_{j\sigma_j}$ are considered from the beginning for all *iϵI*; if in **"reverse"** order, they are considered from the end. This process starts with element e=1 or e=c by calculating the following sums:

If the aggregation is formed in **"forward"** order, the value of the following expression is determined:

$$\sum_{i\in I_j} \frac{X^i_{j\sigma_{j(e=1)}}}{\gamma_i} \tag{4}$$

If the aggregation is formed in **"reverse"** order, the value of the following expression is determined:

$$\sum_{i\in I_j} \frac{X^i_{j\sigma_{j(e=c)}}}{\gamma_i} \tag{5}$$

The calculated sum is compared with the value of σ_j^{max}. If the comparison shows that condition (3) is satisfied, then the aggregation corresponding to e=2 or e=c−1e is analyzed next. In this way, if the verification confirms that condition (3) is satisfied for all aggregations within the combination, the combination is accepted as a solution. If, during the verification, condition (3) is not satisfied for any aggregation, certain additional adjustments are made to the aggregations to ensure that the condition is ultimately fulfilled.

It should be emphasized that such analytical verification can be performed not only on the aggregations themselves but also on their indices, which simplifies the solution process. In the above condition, the ratio $X_{j\sigma_j(e)}^i/\gamma_i$ represents the index ψ_e^i corresponding to the e-th aggregation. Therefore, condition (3) can be expressed as follows:

$$\sum_{i\in I_j} \psi_e^i \leq \sigma_j^{max} \tag{6}$$

The index analysis satisfying condition (6) is carried out as follows:

The index being analyzed in the aggregation is denoted by its sequence number e. If the aggregation is formed in **"forward"** order, then e=1 at the beginning and subsequently increments as e=e+1. If the aggregation is formed in **"reverse"** order, then e=c at the beginning and subsequently decrements as e=e−1.

For the selected index ψ_e^i, the sum $\sum_{i\in I_J} \psi_e^i \leq \sigma_j^{max}$ is calculated and compared with the value of σ_j^{max}.

If the comparison results in $\sum_{i\in I_J} \psi_e^i \leq \sigma_j^{max}$ then the value of index eee is incremented (or decremented), i.e., e=e+1 (or e-1) and the analytical verification process is repeated for the next index.

If the verification results show that condition (6) is satisfied for all e∈{1,2,…,c} then the corresponding combination meets condition (6) and is considered a solution to the problem. If condition (6) is not satisfied for any index eee, the set numbers must be adjusted (restructured) so that condition (6) is fulfilled for the corresponding combination.

Now, let us consider the ways to restructure the set numbers when condition (6) is not satisfied. Suppose it is determined for which index eee the following relationship holds:

$$\sum_{i\in I_j} \psi_e^i > \sigma_j^{max} \tag{7}$$

It can be observed that in the sets, the sum of the indices ψψ_e^i exceeds the value of σ_j^{max}. Therefore, in order to reduce this sum at the given indices, certain shifts must be performed. If the combination of sets is arranged from the "beginning," the shift can only be made to the "right," whereas if it is arranged from the "end," it can only be made to the "left."

The required shift value $\Delta\psi_e^i$ is determined by the following difference:

$$\Delta\psi_e^i = \sum_{i\in I_j} \psi_e^i - \sigma_j^{max} \tag{8}$$

Thus, by performing a shift of ψ_e^i on the indices $\Delta\psi_e^i$ in the set numbers, new numbers can be formed that satisfy condition (6).

If the shift is performed to the "right," it is appropriate to start from the largest value of the iii-index; if to the "left," it is appropriate to start from the smallest value. Consequently, the order of the ψ_e^i indices, designated as the sender, is determined according to the direction of the shift: if the shift is to the "right," the sequence is {I,I−1,…,i,…,1}; if to the "left," the sequence is {1,2,…,i,…,I}. The rationale for such a sequence is based on the following considerations.

A "right" shift of the numbers' indices corresponds to transferring a certain number of **automobile transport vehicles (ATVs)** from shorter-distance routes to longer-distance routes, and such a shift should primarily be applied to ATVs with higher load capacity. Conversely, a "left" shift should appropriately start with ATVs that have lower load capacity.

Next, for the formed combination of sets, the sum $\sum_{i\in I}\sum_{j\in J} X_{j\sigma_j} \cdot Q_{ij}$ is calculated, and it is checked whether this value falls within the range Q_{min}÷Q_{max} If it does, the corresponding numbers are considered a solution to the problem. If, after analyzing all ATVs of type i∈Ii \in Ii∈I in this manner, there remains any part requiring further shifts, the problem does not have a solution.

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