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Mathematical Model of Regulators Adapting to Disturbances

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Abstract. In this article, we consider how the wave-based approach to disturbance modeling, combined with modern control ideas and the concept of state variables, can be used in designing various highly efficient closed loop regulators. These regulators, called "disturbance-adapting regulators," ensure high quality system control over a wide range of both transient and steady disturbances acting on the system. In practice, the last subset often turns out to be empty (sometimes none of the disturbance components can be directly measured). The ability to effectively counteract disturbances even when they cannot be directly measured is one of the most important features of disturbance adapting regulators.

INTRODUCTION

In real control systems, it is rarely possible to obtain real time direct measurements of all system state variables.$\{x_1(t), x_2(t), \ldots, x_p(t)\}$.

Real time direct measurement of all disturbance components$\{\omega_1(t), \omega_2(t), \ldots, \omega_p(t)\}$, is also difficult or sometimes impossible. Thus, the developed regulator theory is based on the assumption that the regulator functions using only three conditions:

1) real time measurements of output variables$\{y_1(t), y_2(t), \ldots, y_m(t)\}$ in equation (1);
2) currently known set points and control (command) signals;
3) real time measurements of a subset of disturbance components. $\omega_i(t)$.

There are three viewpoints a designer may adopt regarding disturbances in control tasks. He may take the usual viewpoint that disturbances negatively influence system behavior. In that case, optimal disturbance suppression is achieved if the regulator is designed to eliminate the influence of any disturbances on the system [1-3].

Thus arise a special class of regulators called disturbance absorbing regulators. In some cases, the structure of the system does not allow full elimination of disturbance influence. Then the designer may instead seek to minimize disturbance effects, leading to disturbance minimizing regulators [4,11].

Finally, the designer may take an optimistic viewpoint that some disturbances can sometimes affect the system beneficially. In this case, optimal regulation is achieved by using all potentially useful disturbance effects — regulators that use disturbances. In some systems, requirements may lead to a combination of these three types, forming multifunctional disturbance adapting regulators [5,6,16].

METHODS

To ensure maximum generality, we begin with a general mathematical model for a multidimensional (possibly time varying) linear system

:$\dot{x} = K(t)x + L(t)u(t) + F(t)w(t),$

$$y = M(t)x + N(t)u(t) + G(t)w(t), \quad (1)$$

where $x = (x_1, x_2, \dots, x_n)$ – is the state vector, $u = (u_1, u_2, \dots, u_n)$ – is the input vector, $\omega = (\omega_1, \omega_2, \dots, \omega_n)$ – is the disturbance vector, and $y = (y_1, y_2, \dots, y_n)$ – is the output vector [17]. The matrices $K(t), L(t), F(t), M(t), N(t)$ и $G(t)$ are assumed to known. Of course, in specific cases, some of these matrices may be constants (or perhaps even bullets). The inclusion of terms in expression (1) for the output variable is explained by the fact that designers often use accelerometers as output sensors [7,8].

Disturbances ω(t)=[ω_1(t)..., ω_p(t)], have wave like structure and can be modeled using the disturbance state model:

$$\omega(t) = P(t)z + Q(t)x,$$
$$\dot{z} = R(t)z + S(t)x + \sigma(t), \quad (2)$$

where $z = (z_1, z_2, \dots, z_p)$is the "disturbance state. Matrices $P(t), Q(t), R(t), and\ S(t)$ in equation (2) are assumed known. The matrices $Q(t)$ and $S(t)$ in equation (2) are zero in most practical cases, but their inclusion in the general model (2) allows the designer to take into account the (rare) cases of the presence of (linearized) "state-dependent disturbances" described in Section. The matrix $P(t)$ is most important since it captures the wave dynamics observed experimentally. $\omega(t)$ The matrix $P(t)$ shows how the various basis functions $\{z_1(t), z_2(t), \dots, z_p(t)\}$ generated by equation (2) are linearly combined to form a system of real components of disturbances $\{\omega_1(t), \omega_2(t), \dots, \omega_p(t)\}$. Similarly the matrices , $F(t)$ and , $G(t)$ in equations (1) show how each component of the disturbance $\omega_i(t)$ enters into the system dynamics equation. As established in Section 2, the disturbance model (2) can effectively describe virtually any disturbance of the wave structure $\omega(t)$ that may be encountered in the design of real control systems [9,10].

RESULT AND DISCUSSION

If disturbances w(t) in (1) have wave structure described by (2), a state estimator can be built that provides real time estimates of the disturbance state z(t). Such an estimator uses measurements of y(t), u(t), and measurable components of w(t). A combined estimator can be constructed to estimate both x(t) and z(t):

$$u(t) = \varphi[\hat{x}(t), \hat{z}(t), t], \quad (3)$$

where $\hat{x}(t), \hat{z}(t)$ *denote the current estimates of x(t)., z(t obtained from a combined state builder that operates on the basis of data on y(t)* from equation (2), the control and *(t),* and any measured disturbance components. Provided that the estimation errors

$\varepsilon_x = x(t) - \hat{x}(t)$ и $\varepsilon_z = z(t) - \hat{z}(t)$ quickly tend to zero compared to the settling time of the entire system, the physically implemented controller (3) is a good engineering approximation of the control law [12,16].

A simple form of a combined state builder. State builders can be designed in various forms. One of the simplest forms of a combined state builder for the system and disturbance model (2) is defined as follows [15,16]. Suppose that the number of components (subset) of disturbances w_i, that can be directly measured is s, and they are denoted by $w_m = (w_{m_1}, \dots, w_{m_s})$; the subset $w_m$is expressed using the common disturbance vector $w_m = (w_1, \dots, w_p)$ in the form

$$w_m = Jw, \quad (4)$$

where $J - (s \times p)$ matrix of rank s. Note that if none of the components of the disturbances w_i is measurable, then we simply need to set$s = 0$, $J = 0$. Now we denote

$$\tilde{G} = \frac{G}{J} \quad (5)$$

Then the combined disturbance plotter for equations (1) and (2) is defined as follows:

$$\left(\frac{\dot{\hat{x}}}{\dot{\hat{z}}}\right) = \left[\frac{K+FQ+A_{11}M+[A_{11}|A_{12}]\tilde{G}Q}{S+A_{11}M+[A_{21}|A_{22}]\tilde{G}Q} \middle| \frac{(F+[A_{11}|A_{12}]\tilde{G})P}{R+[A_{21}|A_{22}]\tilde{G}P}\right] \left(\frac{\hat{x}}{\hat{z}}\right) - \left[\frac{A_{11}}{A_{21}}\right] y - \left[\frac{A_{12}}{A_{22}}\right] w_m + \left[\frac{L+A_{11}N}{A_{21}N}\right] u, \quad (6)$$

where K ,*L, M, R, N, F, G,* $\tilde{G}$, *P, J, Q, S* –the same matrices as in equations (1), (2), and (5). The symbols *y,* w_m and u in equation (6) denote, respectively, the current measured values of the system output variable vector (1), the measured disturbance elements (4), and the control input variable (1). Matrices $A_{11}, A_{12}, A_{21}, A_{22}$ in equation (6) are selected by the designer based on stability considerations discussed below. Block diagram of the combined state builder described by equation (6) [13,15].

The quality of the estimates $\hat{x}(t), \hat{z}(t)$, performed by state builder (49), can be judged based on the analysis of the dynamic behavior of the combined state vector $(\varepsilon_x|\varepsilon_z) = [x(t)|z(t)] - [\hat{x}(t)|\hat{z}(t)]$. From equations (42) and (43) it follows directly that$(\varepsilon_x|\varepsilon_z)$ are described by a system of $(n + p)$ first-order differential equations:

$$\begin{pmatrix} \dot{\varepsilon_x} \\ \dot{\varepsilon_z} \end{pmatrix} = \left[\frac{K+FQ+A_{11}M+[A_{11}|A_{12}]\tilde{G}Q}{S+A_{11}M+[A_{21}|A]\tilde{G}Q} \middle| \frac{(F+[A_{11}|A_{12}]\tilde{G})P}{R+[A_{21}|A_{22}]\tilde{G}P}\right] \begin{pmatrix} \varepsilon_x \\ \varepsilon_z \end{pmatrix} + \begin{pmatrix} 0 \\ \sigma(t) \end{pmatrix}. \quad (7)$$

CONCLUSIONS

Complete absorption of disturbances. To completely eliminate the influence of disturbances $w(t)$ on the behavior of system (1), control u must counteract the influence of the terms $F(t)w(t)$ and $G(t)w(t)$ and simultaneously control the state $x(t)$ [and/or y(t)] in the desired manner. To achieve this, we assume that the overall control action u is split into two parts:

$$u = u_d + u_p. \quad (8)$$

The component u_d is assigned the task of absorbing disturbances $w(t)$, and the component u_d —is assigned the task of the required control of the state $x(t)$ and/or variable $y(t)$ (thus, $w(t)$ performs the main control task). Then, substituting expression (8) into equation (1), we obtain [below, the argument t in the matrices is sometimes omitted for the sake of simplicity of notation];

$$\dot{x} = Kx + Lu_p + Lu_d + Fw(t),$$
$$y = Mx + Nu_p + Nu_d + Gw(t), \quad (9)$$

From equations (9) it follows that for complete absorption of disturbances the value of u_d must be chosen so that the equations are satisfied

$$Lu_d(t) \equiv -Fw(t), \qquad Lu_d(t) \equiv -Gw(t), \quad (10)$$

for all possible disturbance vectors $w(t)$. The range of possible values $w(t)$ is given by equation (2) where $z -$ *arbitarary* p —vector, and $x -$ *arbitarary* $n -$ *vector*. Thus, using equation (2), we can express the projection conditions (60) in the form

$$\left[\frac{L}{N}\right] u_d(t) \equiv -\left[\frac{FP \mid FQ}{GP \mid GQ}\right], \quad (11)$$

where (z/x) arbitrary. A necessary and sufficient condition for the existence of a control u_d. Satisfying equation (11) has the form

$$rank\left[\frac{L \mid FP \mid FQ}{N \mid GP \mid GQ}\right] = rank\left[\frac{L}{N}\right], \quad (12)$$

Further, satisfaction of the criterion of complete absorption (12) means that

$$\left[\frac{FP \mid FQ}{GP \mid GQ}\right] = \left[\frac{L}{F}\right]^{[G]},$$

for some matrix G. The most general form of the "solution" of equation (11) with respect to the control u_d of the absorbing disturbance is given in the work [11]:

$$u_d = -G\begin{pmatrix} z \\ x \end{pmatrix} = -G_1 z - G_2 x, \qquad G = [G_1 \mid G_2], \quad (13)$$

where any matrix from the family can be chosen as G

$$G = \left[\frac{L}{N}\right]^+ \left[\frac{F}{G}\right]^{[P \mid Q]} + \left(I - \left[\frac{L}{N}\right]^+ \left[\frac{L}{N}\right]\right) Q_G. \quad (14)$$

Here Q_G – completely arbitrary matrix of (real) parameters. The symbol $[\,\cdot\,]^+$ in equation (14) denotes the well-known generalized Moore–Penrose inversion of the matrix $[\,\cdot\,]$ usually, the designer chooses the coefficient matrix G such that $\| G \|$ is minimal in some sense. Choosing $Q_G = 0$ in equation (14) leads to a particular solution $G = G_*$, in which each column of the matrix G_* has the minimum possible norm. Note that if

$$rank\left[\frac{L}{N}\right] \equiv r,$$

then (the bar indicates transposition)

$$\left[\frac{L}{N}\right]^+ = \left(\left[\frac{L}{N}\right]' \left[\frac{L}{N}\right]\right)^{-1} \left[\frac{L}{N}\right]',$$

and equation (14) automatically gives the unique solution $G = G_*$.

The calculation of $[\,\cdot\,]^+$ for other cases is described by Kalman and Englar [16].

Thus, the main criterion for achieving complete absorption of disturbances $w(t)$ in model (2) is expressed by equation (12). Equations (13) and (14) define "regulators adapting to disturbances", which ensures complete absorption of disturbances; in practice, equation (13) can be implemented in the form

$$u_d = -G_1\hat{z} - G_2\hat{x}, \qquad G = [G_1|G_2], \tag{15}$$

where $\hat{z},\ \hat{x}$ are generated by the combined state builder.

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