V International Scientific and Technical Conference Actual Issues of Power Supply Systems

Mathematical analysis of pressure and radial forces in centrifugal pumps with respect to fluid density (case of Zarafshan region)

AIPCP25-CF-ICAIPSS2025-00178 | Article

Mathematical analysis of pressure and radial forces in centrifugal pumps with respect to fluid density (case of Zarafshan region)

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Abstract. This paper analyzes the performance of centrifugal pumps with emphasis on fluid density effects. Based on Euler's equations, simplified mathematical expressions were derived to describe pressure rise and pressure distribution in the impeller. The results show that increasing fluid density leads to higher radial forces and pressure rise, which significantly affect pump efficiency and reliability. The proposed model, adapted to the conditions of the Zarafshan region, provides theoretical insights for predicting pump behavior under varying fluid properties.

INTRODUCTION

Centrifugal pumps are widely used in the Zarafshan region for pumping various liquids (water, mineral solutions, petroleum products, etc.). During the operation of the pump, the main forces acting on the impeller are directly related to the liquid density, and the change in these forces affects the reliability and efficiency of the device. It is especially important to conduct a mathematical analysis of this issue under conditions of high temperature and different density indicators of liquids in the region.Determination of pressure, force, and blade rotation frequency depending on the density of the liquid based on the mathematical analysis of centrifugal pumps.

Problems with fluid density change

The pump's load increases. As the density increases, the centrifugal force also increases, resulting in an increase in the mechanical load and pressure on the impeller (wheel). This causes additional vibrations and wear in the bearing and shaft [1,2,3-15].

EXPERIMENTAL RESEARCH

Energy consumption will change. For the discharge of heavier (higher density) liquids, more power is required, which negatively affects the pump's efficiency.

Cavitation risk. A decrease in density or temperature change can accelerate cavitation processes.

Disruption of the pressure regime. Depending on the density, the pressure increase also changes, as a result of which hydraulic stability in the system is disrupted, and unexpected emergency situations may occur.

In the initial case, based on the analysis of energies in the liquid, we express the equality of energies through the following equality. [4,5-30].

$$E_K = E_P \tag{1}$$

We construct the expression for the frequency of the working wheels of the above equation 1 in the following way.

$$\frac{m\vartheta^2}{2} = mgh \Rightarrow \vartheta^2 = 2gh \tag{2}$$

$$\vartheta = \omega r \Rightarrow \omega^2 r^2 = 2gh \text{ if;} \tag{3}$$

$$\omega = 2\pi\nu \Rightarrow 4\pi^2 r^2 \nu^2 = 2gh \Rightarrow \nu^2 = \frac{2gh}{4r^2\pi^2} \Rightarrow \nu = \frac{\sqrt{2gh}}{2\pi r} \tag{4}$$

where: E_k,E_p - kinetic and potential energies of the liquid (J), m - unit mass of the liquid (kg), g - acceleration due to gravity (m/s^2), h - height of the liquid (m), ϑ -liquid velocity (m/s), ω- angular velocity of the impeller (rad/s), ν- rotational speed of the impeller (s^{-1}).

We introduce the blade rotation frequency as a function with the following variables. [6-10,13-30]

$$\nu(r,h) = \frac{\sqrt{2gh}}{2\pi r} \tag{5}$$

Due to the hydrostatic pressure force, we express the pressure through the following formula in terms of the blade rotation frequency and radius.

$$P = \rho \Rightarrow\ \omega^2 r^2 = 2gh \Rightarrow h = \frac{\omega^2 r^2}{2g} gh \tag{6}$$

$$P(\omega, r, \rho) = \rho\frac{\omega^2 r^2}{2} \tag{7}$$

if; $\Delta d \to 0$ R+r≈ $2R$ R-r=Δd

$$F = S(P_1 - P_2) \Rightarrow F = S\left(\frac{\rho\omega^2}{2}R^2 - \frac{\rho\omega^2 r^2}{2}\right) = S\rho\frac{\omega^2}{2}(R^2 - r^2) \tag{8}$$

$$F = S\Delta d\rho\frac{\omega^2}{2}2R \Rightarrow F = \Delta V\rho\omega^2 R \tag{9}$$

$$F(\omega, R) = m\omega^2 R \tag{10}$$

where: P - pressure (N/m), ρ -liquid density (kg/m^3), R - outer radius - the maximum radius of the impeller, the distance to the outlet edge (m), r - inner radius - the area close to the center of the impeller, i.e., the radius of the inlet (m), Δd - radial width of the impeller channel, i.e., -the radial difference in wing length (m), F - force (N), S - area of impact of the impeller (m^2), P_1,P_2 - inlet and outlet pressures (Pascal), ΔV -volume change (m^3)

Newton's second law for an elementary ring inside a centrifugal pump:

$$dF = \rho\omega^2 r dV \tag{11}$$

$$dP = \rho\omega^2 r dr \tag{12}$$

Integrating:

$$P(r) = \frac{\rho\omega^2}{2}(R^2 - r^2) \tag{13}$$

RESEARCH RESULTS

Calculating the forces acting on the impeller of a centrifugal pump based on the density of various liquids found in the Zarafshan region, we construct the following graph using Formula 10.

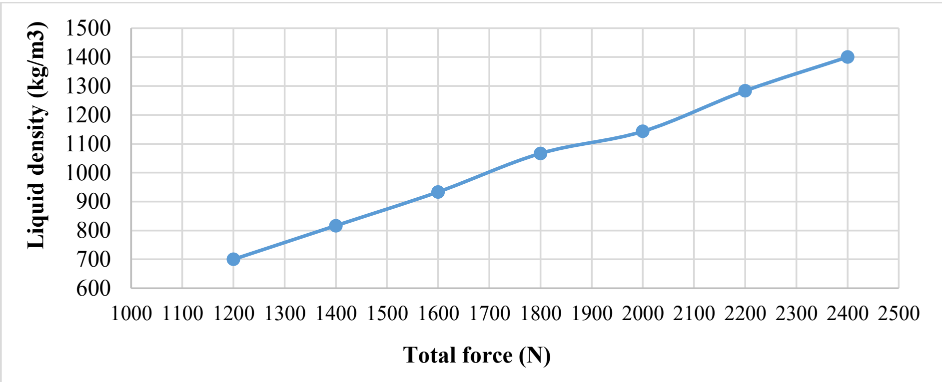

Figure 1. Graph of inertia, partial pressure, and total forces acting on the impeller of the pump to determine the density of the liquid.

Results table (sample):
Pure water (1000 kg/m3): $F_{inertia}$ ≈ 1659 (N), $F_{pressure}$ ≈ 62 (N), F_{total} ≈ 1721 (N).
Light oil (820 kg/m3): $F_{inertial}$≈ 1361 (N), $F_{pressure}$ ≈ 51 (N), F_{total}≈ 1412 (N).
Meltwater (1050 kg/m3): $F_{inertial}$≈ 1745 (N), $F_{pressure}$ ≈ 65 (N), F_{total}≈ 1810 (N).
Salt solution (1200 kg/m3): $F_{inertia}$≈ 1996 (N), F $_{pressure}$ res ≈ 79 (N), F_{total}≈ 2075 (N).

Mineral waters (1300 kg/m3): $F_{inertia}$≈ 2162 (N), $F_{pressure}$ ≈ 86 (N), F_{total}≈ 2248 (N).

The main forces acting on the impeller of centrifugal pumps are directly related to the liquid density . Analytic integrals (radial resultant from inertial force and pressure) show that the components of inertial forces and pressure flow increase linearly in density; as a result, the total radial load

This means that high-density liquids used in the Zarafshan region (for example, saline solutions or mineral liquors) impose significant radial loads on the pump impeller, rudder, and shaft - requiring a re-planning of material selection and service intervals. Based on the density of various liquids found in the Zarafshan region, we calculate the pressures acting on the impeller of the centrifugal pump in ideal and real conditions and construct the following graph using formula 12.

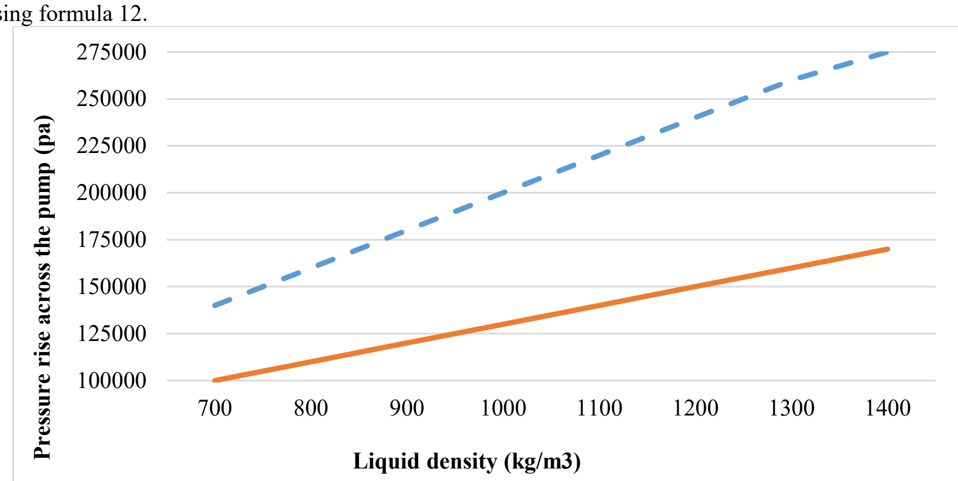

Figure 2. Pressure increases on the impeller of a centrifugal pump depending on the liquid density

The ideal model is based on Euler's formula, calculated without taking losses into account.

Real model - taking into account the coefficients of change in internal and external radii, hydraulic efficiency, and flow losses. Analysis of the graph shows that the actual pressure rise occurs at significantly lower values compared to the ideal model. This indicates the need for structural optimization of pumps when working with liquids of various densities in the Zarafshan oasis. This indicates the need to substantiate the radial pressure distribution in the centrifugal field. It shows how the pressure changes radially inside the pump impeller, we construct the following graph using formula 13 in graph 3 below.

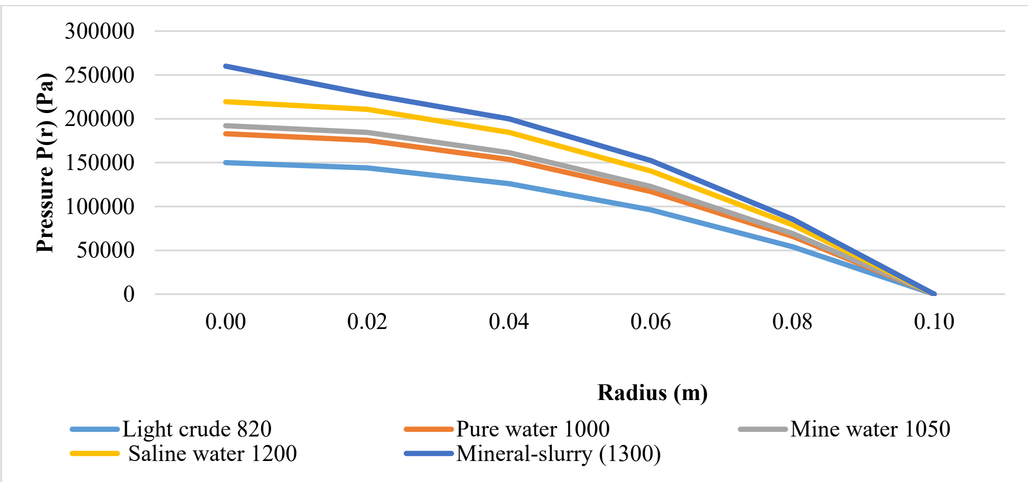

Figure 3. Radial pressure distribution in the impeller of a centrifugal pump

The results presented in Fig. 2 and Fig. 3 above show a clear mathematical relationship between the fluid density and the pressure distribution along the radius in the impeller of the centrifugal pump. As can be seen from Fig. 2, the increase in pressure is directly proportional to the density. This confirms that liquids of different densities (water, saline solution, mineral slurry, etc.), found in the Zarafshan oasis, impose different loads on the pump impeller.

Based on the pressure distribution model in Figure 3 above, it was precisely calculated by radius. This result shows that the pressure is maximal at the center and zero at the outer radius. Also, as the density increases, the entire pressure profile shifts upward.

The novelty of the research is that this mathematical model, combining the parameters of liquid density, radius, and angular velocity, allows for a preliminary assessment of the actual operating modes of pumps used in the Zarafshan oasis. While previous works usually assessed the total hydraulic head or efficiency, in this approach, a precise mathematical expression of the pressure distribution along the radius and density-dependent forces was given. This can serve as a basis for optimizing the design of new pumps by calibrating them with experimental measurements.

In this study, a mathematical modeling of the operation of centrifugal pumps was carried out in the conditions of the Zarafshan oasis. The main attention was paid to the density of the working fluid, and the following results were obtained:

Radial force - density dependence. The obtained results showed that the total radial force acting on the pump impeller is directly proportional to the liquid density. This allows for a preliminary assessment of the pump's additional load in high-density liquids (e.g., brine or mineral suspensions).

Pressure rise - density dependence. It was found that the increase in pressure in the pump increases in density linearly. This means that the pump's height indicators and energy consumption strongly depend on the liquid composition.

Pressure distribution along the radius. The obtained mathematical expression showed a parabolic distribution of pressure along the radius. At the center, the pressure is greatest, and at the outer radius it decreases to zero. This revealed an uneven distribution of forces along the pump channel.

Scientific novelty lies in the fact that in this work, the mathematical model of centrifugal pumps was considered for the first time using a density-based integrated approach. That is, the relationship between the density of the liquid and the radial force, the increase in pressure, and the pressure distribution along the radius were analyzed.

This approach creates a scientific basis for the preliminary assessment and optimization of the efficiency of pumps operating with liquids of various densities in the Zarafshan oasis. The presented graphs and mathematical results will serve as a basis for further experimental studies and practical projects.

Why was the emphasis placed specifically on density?

Density - the main factor factor of centrifugal force ($F \sim \rho$)

The main parameters of the pump (pressure, power, efficiency) are directly proportional to density.

Liquids of varying densities (salt waters, technological solutions, mineral suspensions in mines) are widespread in the Zarafshan oasis. Therefore, it is this parameter that plays the most decisive role in solving regional problems.

CONCLUSIONS

Other parameters (speed, radius, number of wings) are selected by the designer and are stable; however, density is a variable parameter depending on natural conditions and the technological process. Therefore, the mathematical model emphasized density as a novelty. The scientific novelty of the research lies in the fact that in previous models, more emphasis was placed on geometric and velocity parameters, while in our study, the influence of the density of various liquids found in the Zarafshan oasis on the parameters of the centrifugal field was analyzed on a mathematical basis. The conducted research provides a comprehensive mathematical and analytical assessment of the influence of liquid density on the operating parameters of centrifugal pumps used in the Zarafshan region. The results prove that density is one of the dominant factors determining the hydraulic, mechanical, and energy characteristics of pump operation. Mathematical models developed through energy balance, rotational frequency, radial force formation, and pressure distribution provide clear theoretical justification for practical challenges observed in real pumping systems of the region.

The study demonstrated that an increase in liquid density leads to proportional growth in inertial forces and radial hydraulic loads acting on the impeller. This effect significantly increases the mechanical stress on critical components such as the impeller blades, bearings, and shafts. Consequently, higher-density liquids—common in the Zarafshan oasis (saline waters, mineral solutions, brines, technological suspensions)—create elevated operational risks, accelerate wear, and require more frequent maintenance intervals. The results emphasize the necessity for improved material selection, enhanced strength margins, and optimized structural design of pump elements.

The pressure rise inside the pump, derived from Euler's equation and refined through a real-loss model, showed that actual pressure is notably lower than theoretical predictions. This reveals the importance of accounting for hydraulic losses, turbulence, uneven flow distribution, and viscous drag when designing pumps for diverse-density liquids. The pressure distribution model across the impeller radius identified a parabolic profile where maximum pressure forms near the central zone and decreases toward the outlet edge. This uneven pressure field induces nonuniform loading, which can initiate fatigue failures in long-term operation.

A key scientific novelty of the research lies in integrating density as the central variable within the centrifugal field equations—offering a refined interpretation of pump behavior under region-specific conditions. Previous studies emphasized geometric parameters or velocity fields, while this work demonstrates that density variations fundamentally reshape the pump's mechanical load, hydraulic performance, and energy demand. This approach provides a deeper understanding of how environmental and technological factors in the Zarafshan oasis directly influence pump reliability and efficiency.

Overall, the findings create a solid theoretical and analytical foundation for optimizing pump design, selecting appropriate materials, predicting operational risks, and enhancing efficiency when working with liquids of different densities. The developed mathematical models and graphs will serve as a basis for further experimental validation and engineering design improvements in hydraulic transport systems operating under diverse natural and industrial conditions of Uzbekistan.

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