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Sagatov Miraziz[1,a)], Sytnik Oleksandr[2]

[1] Tashkent state technical university named after Islam Karimov, Tashkent, Uzbekistan
[2] Cherkassy State Technological University, Cherkassy, Ukraine

[a)] Corresponding author: informtgtu@mail.ru

Abstract. Stability is one of the most important conditions for ensuring the operability of electric power systems. System stability is a necessary requirement for managing a given system. Therefore, studying the stability of electric power installations is extremely important. The feasibility of using integral equation methods to study the stability of electric power systems is determined by the smoothing properties of integral operators, as well as the ability to construct computational algorithms for solving equations that are optimal in terms of accuracy. Traditional methods for studying the stability of electric power systems, which rely on assessing the stability of solutions to differential equations, are analyzed. A method for studying the stability of an electric power system using its integral model is described. An example demonstrates the advantages of the proposed method.

INTRODUCTION

One of the main tasks in analyzing electric power systems is to assess their stability during a wide variety of disturbances. These disturbances can be shallow, in the form of disturbances to normal operation due to small, irregular deviations of driving forces from their steady-state values (e.g., deviations in generator electromotive force, motor power, etc.). The ability of a system to return to its initial (or close to it) state during shallow disturbances is usually referred to as the static stability of the system. [1, 2]

Disturbances in electric power systems can be sudden and profound (e.g., in the form of short circuits, power line breaks, etc.). They are usually short-lived (in the case of short circuits, damage is eliminated by high-speed relay protection). During such disturbances, significant deviations of the system's operating coordinates from their initial values can occur. Depending on the development of the transient process, which is determined by the initial state, system parameters, and the duration of the disturbing forces, the system may or may not maintain operating stability. The ability of a system to return to equilibrium after sudden, short-term, profound disturbances is called the dynamic stability of the system. [3-5]

Obviously, if a system is dynamically stable, it is also statically stable. The only exceptions are cases where measures are taken to improve static stability that are ineffective for dynamic stability. Formally, the difference between the two concepts of stability is only quantitative.

From a mathematical perspective, studying the stability of an electric power system's state is traditionally reduced to assessing the stability of solutions to the differential equations of its state under the corresponding disturbances, which can be accomplished by integrating such equations, provided that the numerical stability of the integration is ensured. [6]

The stability of the equilibrium state of energy systems in general is usually assessed on the basis of Lyapunov methods [7], the essence of which lies in the study of the stability of the solution of differential equations of state of these systems.

In general, the stability of a system's equilibrium state (the static stability of a system) can be assessed by solving the characteristic equation of a linearized system of differential equations. The numerical values of the roots of the characteristic equation indicate the degree of stability of the system's state (the magnitudes of the real components of the roots) and the nature of transient processes under disturbances of their normal state (aperiodic or oscillatory). This method of stability assessment should be considered particularly promising due to the rapid improvement of computer modeling tools, which are used to implement algorithms for numerical methods of solving equations. [8-10]

The difficulty of directly calculating the roots of characteristic equations has led to the development of special methods for assessing the stability of a system's equilibrium state without directly solving the characteristic equation. When using such indirect methods for assessing the nature of the roots, stability criteria are based on indirect features, necessary and sufficient, or only sufficient stability conditions, which can be established without solving the characteristic equation based on the considerations presented above (stability criteria). The methodology for determining the primary stability criterion is essentially determined by methods for calculating the roots of equations (algebraic or transcendental), with all the potential difficulties involved in solving these equations.

At the current stage of electric power system development, new challenges have emerged, driven by the advent of high-power pulse converters and the widespread use of computers. The specific nature of these new challenges lies in the need to deal not only with continuous signals but also with pulsed signals described by functions with discontinuities of the first kind. Moreover, existing traditional methods for studying modern electric power systems, particularly systems with pulsed elements, are in many cases poorly suited and in some cases simply unsuitable for practical use. Specifically, the reason for these difficulties is that the vast majority of numerical methods for solving ordinary differential equations do not allow for the use of signals without the condition of their continuous differentiability. Consequently, further development of methods for studying electric power systems is possible through the use of other approaches to constructing their mathematical models, namely, the use of integral equations. Since at present, for the study of the stability of systems, theoretical foundations have been developed only for mathematical models presented in differential form, the task of studying the stability of an electric power system according to its integral model naturally arises. [11-16]

EXPERIMENTAL RESEARCH

We will consider a linear electrical circuit, the processes in which are described by a system of Volterra integral equations of the second kind

$$\dot{k}(t)\dot{y}(t)+\int_{t_0}^{t}\dot{K}(t,\tau)\dot{y}(\tau)d\tau=\dot{l}(t)\dot{x}(t)+\int_{t_0}^{t}\dot{L}(t,\tau)\dot{x}(\tau)d\tau+\dot{f}(t,t_0) \tag{1}$$

In equation (1) and further on the following notations are used:

$\dot{y}(t)=(y^1(t),y^2(t),\dots,y^n(t))^T$ – n – dimensional vector of output signals (currents, voltages) of the circuit;

$\dot{x}(t)=(x^1(t),x^2(t),\dots,x^m(t))^T$ – m – dimensional vector of input effects (currents, voltages);

$$\dot{K}(t,\tau)=K_{i,j}(t,\tau),\ i=\overline{1,n},\ j=\overline{1,n};$$

$$\dot{L}(t,\tau)=L_{i,j}(t,\tau),\ i=\overline{1,n},\ j=\overline{1,m};$$

$\dot{K}(t,\tau)$, $\dot{L}(t,\tau)$ – kernels of Volterra integral operators reflecting the dynamic characteristics of the system;

$$\dot{k}(t,\tau)=k_{i,j}(t,\tau),\ i=\overline{1,n},\ j=\overline{1,n};$$

$$\dot{l}(t,\tau) = l_{i,j}(t,\tau),\ i = \overline{1,n},\ j = \overline{1,m};$$

$\dot{k}(t,\tau)$, $\dot{l}(t,\tau)$ – matrix variables, and unless otherwise specified, the matrix $\dot{k}(t,\tau)$ is considered equal to the identity matrix *I*; t_0 – the moment the system starts functioning (control action is applied); *t* – the current moment in time;

$$f(t,t_0) = (f^1(t,t_0), f^2(t,t_0), \dots, f^1(t,t_0))^T$$

– a free term containing all the information necessary for unique finding *y(t)* for all $t \geq t_0$;

We consider the elements of all written matrices and vectors to be sufficiently smooth functions of time so that all transformations are valid (in particular, so that equation (1) has a unique solution *y(t)*).

Equation (1) describes a linear system and has the property of a linear dependence of the solution on the right-hand side of the equation and, consequently, on the input. Expression (1) describes a system that is physically realized, since the value of the output signal at time *t* is determined only by the values of the input at this and the previous time and is independent of subsequent changes in the signal x(t).

Stability over an infinite time interval. Consider the integral model (1), assuming *k(t) ≡ I*:

$$y(t) + \int_{t_0}^{t} K(t,\tau) y(\tau) d\tau = g(t), \tag{2}$$

$$g(t) = l(t)x(t) + \int_{t_0}^{t} L(t,\tau) x(\tau) d\tau + f(t,t_0).$$

Using the resolvent, the solution to the integral equation (2) is written as follows

$$y(t) = g(t) + \int_{t_0}^{t} R(t,\tau) g(\tau) d\tau.$$

Substituting the expression for the function *g(t)* into this formula, after equivalent transformations we find

$$y(t) = y_1(t) + y_2(t), \tag{3}$$

where

$$y_1(t) = \int_{t_0}^{t} w(t,\tau) x(\tau) d\tau; \tag{4}$$

$$y_2(t) = \int_{t_0}^{t} w_f(t,\tau) f(\tau,t_0) d\tau; \tag{5}$$

$$w(t,\tau) = l(t)\delta(t-\tau) + L(t,\tau) + R(t,\tau)l(\tau) + \int_{t_0}^{t} R(t,u)L(t,\tau)du \quad ; \tag{6}$$

$$w_f(t,\tau) = \delta(t-\tau) + R(t,\tau) \quad ; \tag{7}$$

$\delta(t)$ –Dirac delta function.

Let's move on to the basic definitions, using the integral model (1) for the mathematical description of the system with $k(t) \equiv I$. The solution to equation (1), according to formula (3), consists of two components $y(t) = y_1(t) + y_2(t)$, where $y_1(t)$ are the forced oscillations at the system output caused by the input and are determined by formula (4); $y_2(t)$ are the free oscillations, which are uniquely found from expression (5) and depend only on the initial energy reserve and the properties of the system.

Extending the known formulations to the case under consideration, we call system (1)

– stable (asymptotically) if

$$\lim_{t\to\infty} y_2(t) = 0 \tag{8}$$

for an arbitrary admissible function $f(t, t_0)$, which is determined by the initial energy supply in the system;

– non-resonant (stable in the sense of: limited input - limited output), if from the limited nature of the input action $\|x\| < \infty$ it follows that the corresponding response of the system is also limited $\|y\| < \infty$.

Let us now turn to the analysis of the stability conditions. From formula (5) for the function $y_2(t)$ it follows that the stability property mainly depends on the type of the resolvent kernel $R(t, \tau)$ and, to some extent, on the structure of the free term $f(t, t_0)$. The latter feature is characteristic of linear systems with non-degenerate kernels, the equations of which cannot be reduced to differential equations. Consequently, the stability of the system is mainly determined by the properties of the kernel $K(t, \tau)$, which is largely analogous to the traditional case where stability is one-to-one related to the distribution of the poles of the transfer function. In addition, the following statement can be proved: if there exists a time instant $t_1 < \infty$ such that for all $\tau \in [t_1, t]$ the function $K(t, \tau)$ satisfies the conditions

$$\left.\begin{array}{lll} K(t,\tau) \le 0 & at & t \ge t_1, \\ \lim\limits_{t\to\infty} K(t,\tau) < 0, & & \end{array}\right\} \tag{9}$$

and the function $f(t, t1)$ can be chosen to be negative on the interval $[t1, \infty)$ and positive on some set of nonzero measure, then system (1) is unstable.

In fact, let us assume for definiteness that $f(t_1, t_1) > 0$, $f(t, t_1) \ge 0$ and is continuous, and the kernel $K(t, \tau)$ is summable over τ on the interval $[t_1, t]$.

Then, taking $x(t) \equiv 0$ in equation (1) according to the superposition principle, we find

$$y_2(t_1) = f(t_1, t_1) > 0,$$

$$y_2(t) = f(t, t_1) - \int_{t_1}^{t} K(t,\tau) y_2(\tau) d\tau \ge 0$$

for all $t \ge t_1$, since $-K(t, \tau) \ge 0$. In addition, from expressions (9) it follows that

$$\lim_{t\to\infty} y_2(t) > 0 \quad ,$$

which contradicts requirement (8). Consequently, conditions (9) are sufficient conditions for instability and, in general, cannot be weakened, as the example below shows. Note also that for the system to be stable, at least one of the inequalities (9) must be violated.

RESEARCH RESULTS

For the electrical circuit shown in Fig. 1, we will consider the issue of the stability of the equilibrium state of a nonlinear *R, L, C* circuit with nonlinearity of the inductor and resistor.

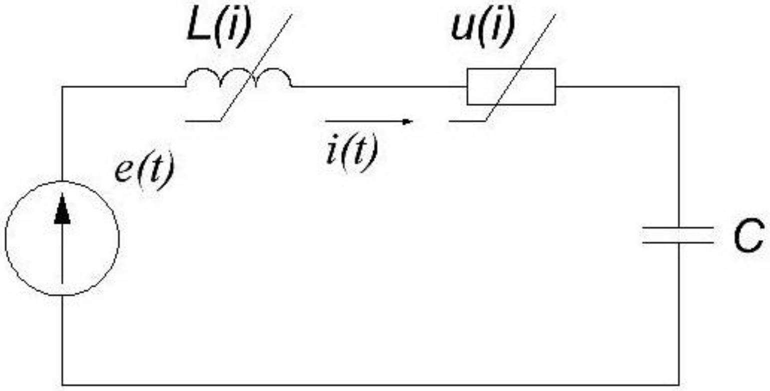

FIGURE 1. Nonlinear *R, L, C* circuit.

The equation of state of this circuit is

$$\frac{d\psi(i)}{di}\frac{di}{dt}+u_r(i)+\frac{1}{C}\int_0^t i(t)dt=e(t) \quad (10)$$

Let us first consider the traditional approach to solving this problem based on the use of a differential model. The linearized, corresponding [7] homogeneous equation will have the form

$$\left(\frac{d\psi(i)}{di}\right)_{i_0}\frac{d\Delta i}{dt}+\left(\frac{du(i)}{di}\right)_{i_0}\Delta i+\frac{1}{C}\int_0^t \Delta i(t)dt=0 \quad (11)$$

We write the differential homogeneous equation in the form

$$\left(\frac{d\psi(i)}{di}\right)_{i_0}\frac{d^2\Delta i}{dt^2}+\left(\frac{du(i)}{di}\right)_{i_0}\frac{d\Delta i}{dt}+\frac{1}{\tilde{N}}\Delta i=0 \quad (12)$$

The roots of the characteristic equation corresponding to equation (12):

$$p_1=-\frac{r}{2L}+\sqrt{\left(\frac{r}{2L}\right)^2-\frac{1}{LC}},\quad p_2=-\frac{r}{2L}-\sqrt{\left(\frac{r}{2L}\right)^2-\frac{1}{LC}}, \quad (13)$$

where

$$r = \left(\frac{du(i)}{di}\right)_{i_0}, \quad L = \left(\frac{d\psi(i)}{di}\right)_{i_0}. \tag{14}$$

For $\psi(i)$ in the form of the fundamental magnetization curve, the dynamic inductance *L* is always positive. Under this condition, the roots of (13) will have a positive real part only at $r < 0$. They can be either real or complex conjugate.

Consequently, the equilibrium state (11) can be unstable only in the decreasing sections of the volt-ampere characteristic *u(i)*. Moreover, as follows from (12), during an oscillatory process, regardless of the value of *r* in such sections, the equilibrium is always unstable. The disturbance of equilibrium can be aperiodic (the roots are real) or oscillatory (the roots are complex conjugate).

Let us solve the stability issues of the equilibrium state of the reduced electric circuit based on the integral dynamic model. First, by integrating the original equation (10), we obtain an equivalent integral equation. The equivalent integral equation will have the form

$$\Delta i(t) + \int_0^t \left[\frac{r}{L} + \frac{1}{LC}(t-s)\right] \Delta i(t) ds = f(t), \tag{15}$$

where

$$f(t) = \frac{1}{L}\int_0^t e(s)ds,$$

r and *L* – are determined by expression (14);

$$K(t-s) = \left[\frac{r}{L} + \frac{1}{LC}(t-s)\right]$$

– kernel of the integral equation.

The kernel

$$K(t-s) = \left[\frac{r}{L} + \frac{1}{LC}(t-s)\right]$$

will be positive for all $t \geq s$, and also

$$\lim_{t \to \infty} K(t,s) \geq 0$$

for any fixed $s \leq t$ only when $r > 0$. Thus, both instability conditions of the system (9) are violated, meaning that the equilibrium state of the circle can be unstable only in the decreasing sections of the volt-ampere characteristic *u(i)*.

CONCLUSION

Thereby, the considered method of studying the stability of electric power systems using their integral models allows us to significantly expand the class of electrical and electronic circuits under study, including automatic control systems with electronic pulse elements.

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