Selective Pole Placement in Linear Electrical Systems Using Matrix Canonicalization

Tokhir Makhmudov

Tashkent state technical university named after Islam Karimov, Tashkent, Uzbekistan

[a)] *Corresponding author:* t.maxmudov@tdtu.uz

Abstract. This paper presents an enhanced methodology for selective pole control in linear time-invariant electrical systems using matrix canonicalization. Modern electric power systems require advanced mathematical tools capable of modifying only a subset of dominant poles without disturbing the remaining dynamic characteristics. The proposed approach establishes the conditions under which partial pole relocation is achievable and provides a constructive method for synthesizing a family of controllers ensuring the desired eigenvalue placement. Simulation results for a synchronous generator equipped with an automatic voltage regulator demonstrate the effectiveness of the method and confirm the preservation of non-target eigenvalues. The findings are relevant for designing optimal control strategies, improving system stability, and developing advanced excitation systems.

INTRODUCTION

Contemporary electric power systems (EPS) exhibit considerable complexity, featuring sophisticated control architectures and pronounced dynamic interactions among their components. Central to electricity generation, synchronous generators are integrated with high-performance Automatic Voltage Regulators (AVRs) to ensure rapid and precise voltage control. These regulators substantially expand the domain of stable operation and influence the damping of power and voltage oscillations [1]. The effectiveness of such controllers is explained by their ability to impact the generator parameters in real-time and shape the required properties of transient processes [2].

Given the increasing structural complexity of EPS, the growth of distributed generation, the emergence of long power transit flows, and the implementation of market control mechanisms, the requirements for stability and regulation quality are increasing [3]. In this regard, methods for the analysis and synthesis of control devices continue to actively evolve. Particular attention is paid to methods based on the state-space representation of the system, which enables the application of matrix control technologies [4].

Over the past three decades, a significant number of pole placement methods and their modifications have been developed for multivariable linear systems. The number of such approaches exceeds several hundred, including the classical Kalman method [2], modal control techniques [3], methods based on stratification and matrix zeros [4], as well as newer approaches utilizing numerical stability methods and pseudospectral analysis [5].

The problem of selective pole assignment (selective pole shifting) is highly relevant for the synthesis of control regulators [6]. In many practical situations, it is necessary to change only the dominant poles that determine the quality of transient processes, while the remaining eigenvalues are preserved without alteration. The reasons for this problem formulation are as follows [7]:

The system's dynamics are predominantly determined by a small group of poles.

The correction of all parameters in a complex system is redundant and complicates the regulator's implementation [8].

In the case of partial controllability, it is impossible to shift all poles by definition [9].

This paper examines the necessary and sufficient conditions under which a selective shift of a subset of the poles of a linear time-invariant system is possible without changing the others [10-12]. To solve this problem, the canonicalization method is used, which allows for the synthesis of a set of controllers that satisfy the required dynamic characteristics of the system. Furthermore, practical calculations are performed for a synchronous generator model with an AVR, based on the equations presented in [13-15], demonstrating the effectiveness of the proposed approach.

SELECTIVE POLE ASSIGNMENT PROBLEM

Pole placement in a linear time-invariant system is traditionally performed under the assumption of complete controllability. When this condition is met, all eigenvalues of the closed-loop system can be arbitrarily assigned [16]. However, engineering applications often require only a subset of these eigenvalues to be shifted. Dominant poles, which govern the speed and damping of transient responses, typically require adjustment, whereas the remaining poles may remain unchanged [17].

Selective pole control is feasible only when certain algebraic conditions are satisfied. The canonicalization method allows reformulating the system representation such that the influence of the controller on each pole becomes explicitly characterized through matrix stratification. This method ensures the ability to reposition a specified eigenvalue λ to a desired location $\lambda_{desired}$ without altering the non-target eigenvalues. The general controller structure includes degrees of freedom, enabling additional tuning without affecting the relocated poles [18].

Consider a system represented by a dynamic object described in the state-space form as in [19].

$$px(p) = Ax(p) + Bu(p) + x_0, \tag{1}$$

where x(p) is the n-dimensional state vector; u(p) is the s-dimensional control vector; A and B are numerical matrices determining the system's properties; and K is the static controller (gain matrix) in the feedback loop.

$$u(p) = -Kx(p). \tag{2}$$

Let Λ_n denote the complete set of eigenvalues of matrix A, corresponding to the poles of the original system, where each element may, in general, assume complex values. Within this set, let us identify a subset Λ_n, consisting of g poles that require reassignment, while the complementary subset Λ_{n-g} contains the remaining n – g poles whose positions are to remain unchanged [20].

The following assumptions are adopted in accordance with [4].:

- the poles belonging to Λ_n that are subject to relocation are known in advance, whereas knowledge of the remaining poles is not required.
- none of the poles in Λ_n appears with multiplicity in the complementary set Λ_{n-g}, which ensures the condition $\Lambda_g \cap \Lambda_{n-g} = \emptyset$.

Under these assumptions, the problem considered here is to establish the necessary and sufficient conditions that guarantee the ability to shift the poles in Λ_n to arbitrarily prescribed locations, despite the fact that the poles in Λ_{n-g} remain unknown. In addition, the task includes the synthesis of all controllers capable of achieving such selective pole relocation. Importantly, the eigenvalues belonging to the subset Λ_{n-g} are not required for solving the problem [6].

To address this issue, we draw upon the result established in [3]. Specifically, for an arbitrary linear dynamic system of the form (1), any known real pole λ can be reassigned to a new desired location $\lambda_{desired}$ without affecting the remaining poles. This is accomplished via the state-feedback law (2), constructed using a controller that belongs to the following class of admissible regulators:

$$\{K\}\eta = ((\Theta B^R)^{\sim}(\lambda - \lambda_{desired}) + \overline{\Theta B}^{R} \eta)\Theta, \tag{3}$$

if for the stratification matrix Θ, calculated by the formula

$$\Theta = \overline{A - \lambda I_n}^{L}, \tag{4}$$

the condition is satisfied

$$\Theta B \neq 0. \tag{5}$$

Here η is an arbitrary matrix of size (s – rank(ΘB))×1.

RESULTS AND DISCUSSION

The analysis is conducted on a simplified electrical system.

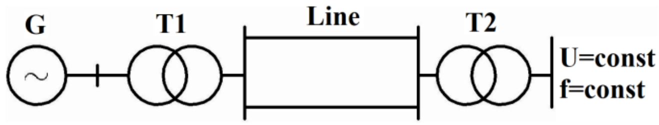


FIGURE 1. Schematic representation of the electrical system.

This study utilizes the mathematical model of the controlled electric power system presented in [5], considering the configuration that includes an automatic voltage regulator. The analysis is carried out under the simplifying assumption that the time constants of the converter, measuring unit, and amplifier are negligible, i.e., $T_c=T_m=T_a=0$. Under these conditions, the system of equations describing the EPS can be reduced to the following form:

$$T_j(d^2\Delta\delta / dt^2) = -P_d(d\Delta\delta / dt) - \Delta P, \tag{6}$$

$$T_{do}(\Delta E'_q / dt) = \Delta E_{qe} - \Delta E_q. \tag{7}$$

The deviation of the EMF will be represented as:

$$\Delta E_q = k_{0\delta}\Delta\delta + k_{1\delta}\frac{d(\Delta\delta)}{dt} + k_{0u}\Delta U_G. \tag{8}$$

Differential equation (7) may be expressed in the following form [6]:

$$T'_d(\Delta E_q / dt) = \Delta E_{qe} - \Delta E_q, \tag{9}$$

where T'_d represents the time constant of the synchronous generator's excitation winding when the stator winding is short-circuited.

The following notations are introduced:

$$\Delta\delta = x_1;\ \frac{d\Delta\delta}{dt} = x_2;\ \Delta E_q = x_3.$$

Taking into account the introduced designations and taking into account (8) and (9), equations (7) and (8) have the form:

$$\begin{aligned} \frac{dx_1}{dt} &= x_2, \\ \frac{dx_2}{dt} &= -k_1x_1 - k_2x_2 - k_3\Delta U_G, \\ \frac{dx_3}{dt} &= -k_4x_3 + k_4\Delta E_{qe}, \end{aligned} \tag{10}$$

$$k_1 = \frac{1}{T_j}(c_1 + b_1k_{0\delta}),\ k_2 = \frac{1}{T_j}(P_d + b_1k_{1\delta}),\ k_3 = \frac{1}{T_j}b_1k_{0u},\ k_4 = \frac{1}{T'_d}.$$

By incorporating the regularizing equation, we obtain:

$$\begin{aligned} \dot{x} &= Ax + Bu, \\ y &= Cx, \\ u &= u, \end{aligned} \tag{11}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -k_1 & -k_2 & 0 \\ 0 & 0 & -k_4 \end{bmatrix},\ B = \begin{bmatrix} 0 & 0 \\ -k3 & 0 \\ 0 & k4 \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix},\ y = \Delta E_q = \begin{bmatrix} \Delta\delta \\ \dfrac{d(\Delta\delta)}{dt} \\ \Delta E_q \end{bmatrix},\ u = \begin{bmatrix} \Delta U_G \\ \Delta E_q \end{bmatrix}.$$

The initial parameters of the electrical power system are defined as follows: $\delta_0=60^0$; $U_c=0.99$; $E_q=1.88$; $P_d=1$; $x_d=2.095$; $x'_d=0.29$; $x_c=0.12$; $T_j=6$sec.; $T_{d0}=4.35$sec. The automatic voltage regulator is configured using the tuning coefficients $k_{0u}=25$; $k_{0\delta}=3$; $k_{1\delta}=2$, which represent the corresponding gains in the voltage and rotor-angle control channels.

By solving the system of equations (10) we obtain a matrix of coefficients:

$$A=\begin{bmatrix}0 & 1 & 0\\ -0.4135 & -0.375 & 0\\ 0 & 0 & -5.88\end{bmatrix};\ B=\begin{bmatrix}0 & 0\\ -1.42 & 0\\ 0 & 5.88\end{bmatrix}$$

with eigenvalues λ_1=–5.78; $\lambda_{2,3}$=–0.1775±0.6251j.

Assume that the pole λ_1 = –5.78 is to be relocated to the desired position $\lambda_{desired}$ = –7.3.

The corresponding matrix can then be calculated as follows:

$$A-\lambda_3 I_3 = \begin{vmatrix}5.88 & 1 & 0\\ -0.4135 & -5.505 & 0\\ 0 & 0 & 0\end{vmatrix}.$$

By canonization of this matrix one can define

$$\Theta = \overline{A-\lambda_3 I_3}^{L} = [0\ \ 0\ \ 1];\ \Theta\text{B}=[0\ \ 5.88];$$

$$\overline{\Theta\text{B}}^{R} = [1;0];\ \Theta\text{B}^{R} = [0;0.1701].$$

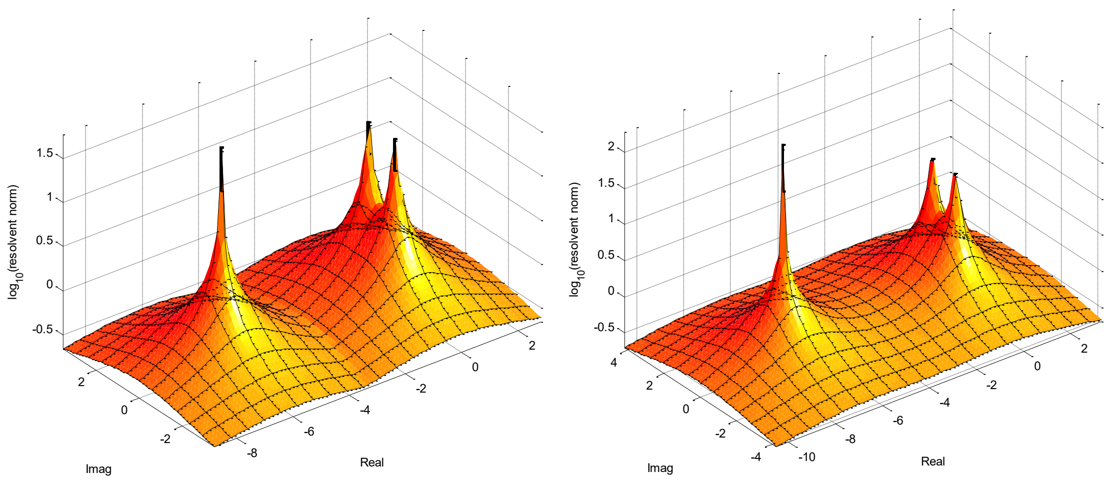
Condition (5) is satisfied, and the formula for the controller (3) gives the following set of values

$$\{K\}\eta = ((\Theta B^{R}) \sim \ \ (\lambda_1 - \lambda_{desired}) + \overline{\Theta B}^{R}\ \eta)\Theta = \begin{vmatrix}0 & 0 & \eta\\ 0 & 0 & 0.19\end{vmatrix}, \qquad (12)$$

where η is any real number. Varying the parameter η changes the system's zeros without affecting the position of its poles.

Direct calculations can verify that any of the controllers (12) provides the desired eigenvalues in the system: λ_1 = –7.3; $\lambda_{2,3}$ = –0.1775 ± 0.6251j.

Presented here is a three-dimensional visualization of the pseudospectrum of matrix A, obtained using the EigTool module [7] developed at Oxford University in conjunction with Matlab. The horizontal plane corresponds to the real and imaginary axes of the complex plane, while the vertical axis depicts the logarithm of the norm of the resolvent function. The peaks in this representation indicate the locations of the matrix’s eigenvalues.



a) b)

FIGURE 2. 3D visualization of the pseudo-spectrum of the electrical system model:
a) initial electrical system model; b) system model after displacement of one pole.

A three-dimensional representation of the pseudospectrum of matrix A is presented here. The visualization was produced using the EigTool module [7] in the Matlab environment, which allows one to examine how the eigenvalues of a matrix behave under small perturbations. In the figure, the two horizontal axes correspond to the real and imaginary parts of the complex plane, while the vertical axis shows the logarithm of the resolvent norm. The elevated peaks on the surface clearly mark the locations of the eigenvalues, making it easy to see how they are distributed and how sharply they are defined.

CONCLUSIONS

The findings of the research indicate that advanced matrix-oriented analytical frameworks—particularly stratification and system canonization—provide an effective foundation for addressing the problem of selective pole manipulation in linear time-invariant systems. The study establishes that, as long as controllability requirements are met and the pole configuration is suitable, the target eigenvalues can be reassigned while leaving the remaining dynamic characteristics of the system unchanged.

Furthermore, the paper demonstrates that the canonization procedure makes it possible to construct an entire class of controllers capable of enforcing the prescribed pole relocation. Importantly, these controllers retain internal parameter freedoms that do not affect eigenvalue placement. This structural flexibility allows for further refinement of the controller with respect to a broad range of performance metrics, including energy consumption, control-effort constraints, oscillation suppression, and others.

Computational experiments carried out for a power-system model incorporating a synchronous generator and an automatic voltage regulator substantiate the theoretical results. Adjusting the dominant pole led to a measurable improvement in dynamic behavior and an increase in stability margins, all while preserving the structure of the remaining modal responses. Visualization of the pseudospectrum using the EigTool package revealed the displacement and clustering of eigenvalues under the influence of the synthesized controller.

The outcomes of this work can be applied to the development of automatic excitation control systems, the design of optimal regulators for large-scale electric power grids, and the stability assessment of systems featuring distributed generation. The proposed methodology also offers a basis for future research, including potential extensions to nonlinear dynamics and stochastic models.

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