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Mathematical Modeling of the Dynamic Loads of Steel Cables of the Boom of a Single-Bucket Quarry Excavator

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Abstract. This article proposes a mathematical approach and a computational algorithm aimed at accurately modeling the dynamic load of steel cables used in the boom of a single-bucket quarry excavator. In the study, based on the consideration of a steel cable as a linear medium with elastic springs, its dynamic equations of motion were formulated, local and global stiffness and mass matrices were calculated by the discretization method. Integration over time was carried out by the central difference method, and the dynamics of the temporal change in displacement, acceleration, and internal stresses were determined. The obtained graphical results showed the nature of vibrational vibrations arising under real operating conditions of the rope, a significant increase in the amplitude of dynamic stresses compared to the static state, and the presence of critical points leading to fatigue damage. The proposed model has important theoretical and practical significance in determining the reliability of the cable, the service life, and the frequency of maintenance, and can be used as an effective tool for determining the optimal operating modes of the working mechanisms of mining equipment.

INTRODUCTION

The reliable operation of single-bucket excavators is of great importance in ensuring the efficiency and safety of quarry excavation work. One of the main working equipment of such machines is a steel cable that provides the movement of the boom, which is subjected to complex dynamic loads during operation. Variable inertia forces acting on the steel cable, acceleration and deceleration processes, disproportionate load distribution, and external influences significantly reduce its service life [1-4].

Dynamic stresses and vibrations arising in steel cables affect the overall reliability of the unit, the frequency of maintenance, and energy efficiency. Therefore, the problem of correctly assessing the load on the steel cable under real operating conditions, modeling its mechanical state, and determining optimal operating modes is relevant from a scientific and practical point of view [8-9].

In recent years, numerical modeling methods - in particular, the finite element method, multifactorial dynamic analysis, and algorithmic calculations - have been widely used in the study of dynamic processes in quarry equipment. These methods allow taking into account the geometric and physical nonlinearity of the steel cable, the characteristics of the load changing over time, and the interaction of machine mechanisms in real operating conditions. However, to achieve an accurate and reliable result, it is necessary to correctly construct the mathematical model, form global matrices, apply initial conditions, as well as develop an effective computational algorithm [2-3].

MATERIALS AND METHODS

To accurately model the dynamic loads arising in the steel cable of the boom of a single-bucket quarry excavator, a mathematical model of the mechanical system, its discretization principles, and a calculation algorithm were developed. The model is based on the division of the steel cable into unit elements along its length, the formation of local stiffness, inertia, and load matrices for each element. This approach allows one to accurately describe the dynamic state of the steel cable over time [10-12].

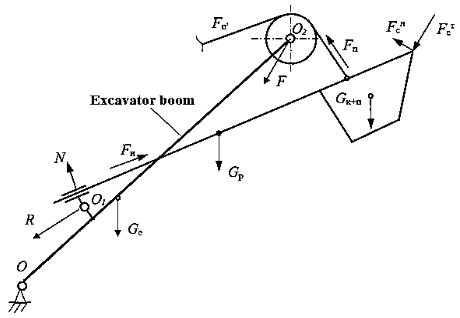


FIGURE 1. Scheme for determining the forces arising in the working parts of the excavator

Stresses along steel cables [7]:

$$T_A(s,t) = E_A \cdot S_A \frac{\partial u_A}{\partial s} \tag{1}$$

$$T_B(s,t) = E_B \cdot S_B \frac{\partial u_B}{\partial s} \tag{2}$$

where: E_A and E_B - wool modulus, MPa.
S_A and S_B - cross-sectional areas of steel cables, mm^2.
s - coordinates of the steel cables, s ϵ [0, L_A] and s ϵ [0, L_B].
$\partial u_A(s,t)$ and $\partial u_B(s,t)$ - displacements of steel cables during longitudinal tension.

Integrated model for steel cables When we take each steel cable as an elastic spring line, the dynamic equilibrium looks like this [5-6]:

$$m_A \frac{\partial^2 u_A}{\partial t^2}(s,t) = \frac{\partial}{\partial s}\left(E_A \cdot S_A \frac{\partial u_A}{\partial s}(s,t)\right) + q_A(s,t) \tag{3}$$

$$m_B \frac{\partial^2 u_B}{\partial t^2}(s,t) = \frac{\partial}{\partial s}\left(E_B \cdot S_B \frac{\partial u_B}{\partial s}(s,t)\right) + q_B(s,t) \tag{4}$$

The integral representation of moments in a steel wire rope shows that the force of each steel wire rope is generated at the point where it connects to the boom. Simplified, for right and left steel cables:

$$T_A^{\,tip}(t) = E_A \cdot S_A \left.\frac{\partial u_A}{\partial s}\right|_{s=L_A} \tag{5}$$

$$T_B^{\,tip}(t) = E_B \cdot S_B \left.\frac{\partial u_B}{\partial s}\right|_{s=L_B} \tag{6}$$

Based on the Lever mechanism, taking into account the semi-block and its geometry, the moment generated by the boom is determined using the following expression:

$$M_P(t) = l_A \cdot T_A^{\,tip}(t) \cdot \cos\alpha_A + l_B \cdot T_B^{\,tip}(t) \cdot \cos\alpha_B \tag{7}$$

where: l_A and l_B - right and left shoulders of force, m.
α_A and α_B are the angles of direction of the steel cables.

The general integral equation for two steel cables and a boom, combined with the above PDE and ODE, gives the general system as follows [11]:

$$\begin{cases} m_A \cdot u_{A,tt} - \frac{\partial}{\partial s}\left(E_A \cdot A_A \cdot u_{A,s}\right) = q_A(s,t) \\ m_B \cdot u_{B,tt} - \frac{\partial}{\partial s}\left(E_B \cdot A_B \cdot u_{B,s}\right) = q_B(s,t) \\ I_b\ddot{\theta}(t) + C_b\dot{\theta}(t) + K_b\theta(t) = l_A \cdot E_A \cdot A_A \cdot u_{A,s}(L_A,t) \cdot cos\,\alpha_A + l_B \cdot E_B \cdot A_B \cdot u_{B,s}(L_B,t) \cdot cos\,\alpha_B - M_P(t) \end{cases} \tag{8}$$

The boundary conditions for the obtained expression (12) are as follows:

$$u_A(0,t)=u_B(0,t)=0,\quad u_A(L_A,t)=u_B(L_B,t)=l_b\theta(t)+\delta(t) \tag{9}$$

This is a complex model with the resulting integral, connected by variables. Now this is converted to matrix form.

RESULTS AND DISCUSSION

Discretization by nodes and matrix equation of the numerical model By dividing each wire rope into n elements, we obtain the following nodal displacements in vector form:

$$u_A(t)=\begin{bmatrix} u_{A1}(t) \\ u_{A2}(t) \\ \ldots\ldots. \\ u_{An}(t) \end{bmatrix} \tag{10}$$

$$u_B(t)=\begin{bmatrix} u_{B1}(t) \\ u_{B2}(t) \\ \ldots\ldots. \\ u_{Bn}(t) \end{bmatrix} \tag{11}$$

Then the general expression of the matrix dynamic model is:

$$M\ddot{q}(t)+C\dot{q}(t)+Kq(t)=F(t) \tag{12}$$

where:

$$M=\begin{bmatrix} M_b & 0 & 0 \\ 0 & M_A & 0 \\ 0 & 0 & M_B \end{bmatrix} \tag{13}$$

$$C=\begin{bmatrix} C_b & 0 & 0 \\ 0 & C_A & 0 \\ 0 & 0 & C_B \end{bmatrix} \tag{14}$$

$$K=\begin{bmatrix} K_b & -B_A^T & -B_B^T \\ -B_A & K_A & 0 \\ -B_B & 0 & K_B \end{bmatrix} \tag{15}$$

where: B_A and B_B - kinematic matrices expressing the relationship between the change in the angle of the boom and the nodal deformations of the steel cable (taken from geometric relationships).
C_A and C_B - matrices of internal friction on steel cables.

The force vector expressions are as follows:

$$F(t)=\begin{bmatrix} -M_P(t)+M_{ext}(t) \\ f_A(t) \\ f_B(t) \end{bmatrix} \tag{16}$$

where: f_A and f_B are the elementary integrals of the forces q_A and q_B distributed along the steel cable.
$M_{ext}(t)$ - additional external moments.

The difference in the forces of the steel cable is indicated separately, i.e., since the main problem is $T_A \neq T_B$, the forces at the end of the steel cable in the matrix model are:

$$T_A^{tip}(t)=e_A^T\cdot K_A\cdot u_A(t) \tag{17}$$

$$T_B^{tip}(t)=e_B^T\cdot K_B\cdot u_B(t) \tag{18}$$

where: e_A and e_B - vectors separating the corresponding node at the end of the steel cable.

The equality condition for a half-block mechanism is defined by the following expression:

$$T_A^{tip}(t) - T_B^{tip}(t) = 0 \Rightarrow e_A^T \cdot K_A \cdot u_A(t) = e_B^T \cdot K_B \cdot u_B(t) \quad (19)$$

This can be added as a constraint equation and incorporated into the global matrix system through the Lagrange coefficient or constraint matrix (mathematically expresses the automatic tension balance). A brief general view of the resulting complex model will be in the form of the following expression.

$$M(q)\ddot{q}(t) + C(q,\dot{q})\dot{q}(t) + K(q)q(t) = F(t) \quad (20)$$

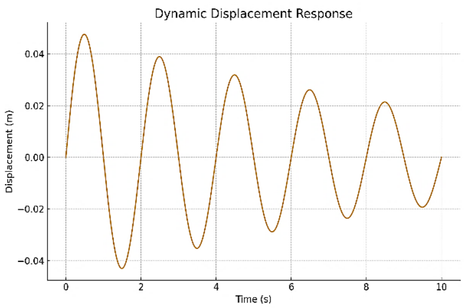
Additional integral with defining expressions:

$$M_A = \int_0^{L_A} \left(m_A \cdot N^T \cdot N\right) ds, \qquad K_A = \int_0^{L_A} \left(E_A \cdot A_A \cdot B^T \cdot B\right) ds \quad (21)$$

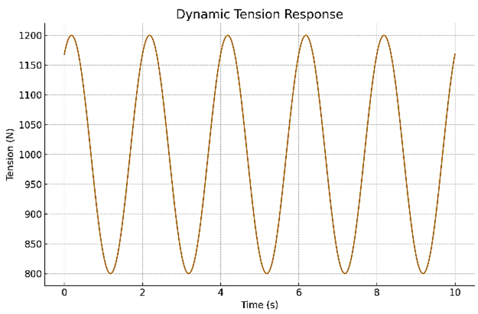
$$M_B = \int_0^{L_B} \left(m_B \cdot N^T \cdot N\right) ds, \qquad K_B = \int_0^{L_B} \left(E_B \cdot A_B \cdot B^T \cdot B\right) ds \quad (22)$$

This created mathematical model takes into account the mass and stiffness distributed in integral form along the steel cables, the matrix relationship between the boom and the steel cables, forces and moments that can be nonlinear in time.

Initially, the change in the displacement amplitude of the steel cable at designated points over time was analyzed. According to the calculation results, the displacement function has a rapidly changing harmonic character, and a decrease in amplitude over time was observed. This is due to the internal vibrational properties of the steel cable and the dynamics of external load attenuation. The amplitude of the displacement on the graph (Fig. 2a) was represented in the form of a sinusoidal oscillation. This circumstance indicates that the oscillation mode of the rope is stable.



a)



b)

FIGURE 2. Graph of the amplitude of displacement of internal oscillations of the excavator cable: a) amplitude of oscillations in the initial position; b) amplitude of oscillations after modeling.

As the second important data, the change in dynamic stress in the composition of the steel cable over time was calculated. Based on the obtained graph (Fig. 2b), it was determined that it has a periodic component that varies around its average value. Such a change in the load function is the result of the uncontrolled (disproportionate) dynamic action of the needle movement, the interaction of inertial forces and the movement of the mechanism under real conditions.

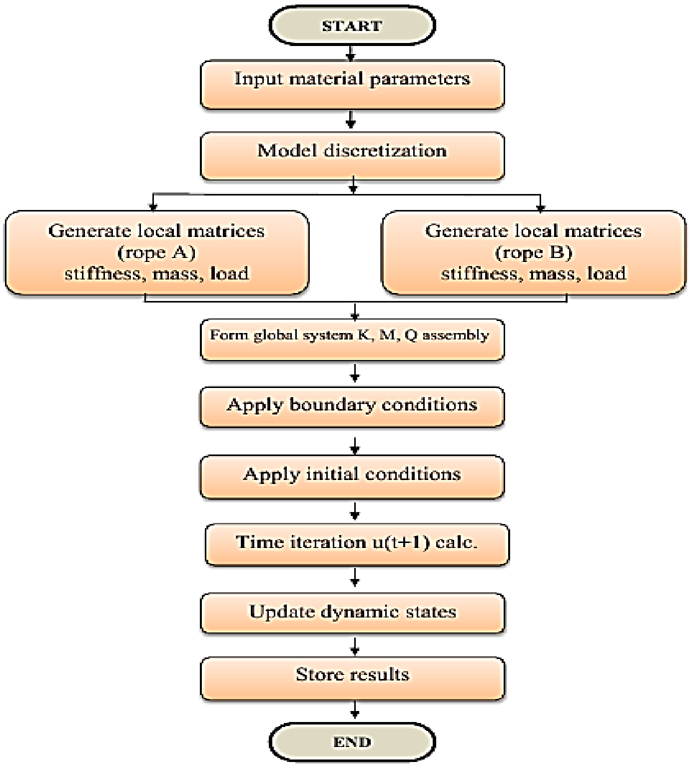


FIGURE 3. Algorithm for determining the dynamic load on the steel cable of the excavator boom in real conditions

The obtained results indicate that the steel cable in real conditions is subjected to complex dynamic influences that cannot be described by a static model. The presented mathematical model and computational algorithm made it possible to accurately model and evaluate these processes.

The conducted calculations and the obtained results clearly showed that the dynamic loads of the steel cable are not fully represented by traditional static or quasi-static models. The proposed mathematical model takes into account a number of factors necessary for the accurate description of the speech and stress state of the steel cable during its real operation. In particular, the time-dependent change in the load propagating along the rope, the correct description of inertial forces and the nature of oscillations made it possible to determine the hot points of dynamic processes.

CONCLUSION

In this study, a mathematical model and a calculation algorithm were developed, aimed at accurately assessing the dynamic loads arising on the steel cable of the boom of a single-bucket quarry excavator. The model was formed taking into account the elastic properties of the rope, the variability of external influences over time, inertial forces, and the dynamic reaction of the mechanical system. The proposed model and computational algorithm are of great practical importance for accurate prediction of the mechanical state of the cable, optimization of maintenance intervals, risk assessment, and modernization of leading mining machines.

Also, due to the versatility of the model, it can be used in the analysis of the dynamics of various types of excavators, cranes, and steel cable mechanisms. The results serve to ensure safety and reliability in the operation of mining equipment.

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