Solving the integrated production planning and scheduling problem via a new hybrid meta-heuristic algorithm

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Abstract. The subject of integrating production planning and control scheduling problems is becoming increasingly important, getting more and more attention in the complicated and constantly changing environment, presenting a new integrated model for aggregate production planning and scheduling problems, in which the production cost, total production time, changeover cost, and product delivery delays are minimized. After that, a new hybrid metaheuristic algorithm is proposed that combines the Whale Optimization Algorithm and the Grey Wolves Optimization Algorithm. The results are compared with different sample sizes using the standard algorithms (Genetic, Whale, and Greywolf) and dynamic programming based on mean square error. The results show that the hybrid meta-heuristic algorithm provides the best results.

Keywords: Whale algorithm, Greywolf algorithm, Aggregate Production Planning, integrated model, Genetic algorithm

INTRODUCTION

Manufacturing organizations rely heavily on production planning, scheduling, and sequencing for optimal performance [1]. Production planning and scheduling represent two essential decision-making tiers within the industrial system. They are critical for decreasing costs, shortening manufacturing cycles, and increasing efficiency [2,3]. In addition, Production planning and scheduling represent two essential decision-making tiers within the industrial system. Managers understand that shifting client expectations can have a substantial economic impact on a company's labour and production decisions. Aggregate Production Planning (APP) refers to strategizing production volumes and scheduling over a medium-term horizon of 3 to 18 months. During this time, production levels are set to meet expected demand. The goal of APP is to set overall production levels to meet the future needs of each product category, even when those needs are not always clear or consistent. APP also evaluates policy and decision-making elements related to appropriate hiring levels, overtime, layoffs, backorders, subcontracting, and inventory management [4,5].

On the other hand, Scheduling requires assigning machines to jobs so that all tasks are completed within the time restrictions. To minimize the objective function, find the most efficient order for executing these jobs on each computer. The main goal is to allocate one or more resources throughout time for activities. Single-machine scheduling is a well-studied type that breaks complicated machine environments into simpler challenges [6]. Because of this, the problem of integrating production planning and scheduling is getting more and more attention in the intricate and changing economic environment. An integrated model that sparked the study of this problem is what led to this [7,8]. Shao et al. [9] suggested that production planning and scheduling go hand in hand and that combining them can significantly increase how well production is managed. Lasserre [10] defined a complete integrated production planning and scheduling model and solved it hierarchically. Xiong et al. [11] examined challenges with multi-period, multi-workshop production planning and scheduling and came up with a way to optimize both at the same time using nonlinear mixed integer programming. Li et al. [12] claimed that using integrated optimization in planning and scheduling can make it easier to make good decisions about how to run processes. They also suggested a new way to break things down using two-level optimization. Based on the State-Task Network (STN) representation, Wangetal [13] created a two-level integrated model for planning and scheduling production. They also came up with the rolling horizon optimization method, which repeatedly solves the bi-level integrated model. The proposed paradigm is both possible and helpful, as shown by examples of benchmarks. Vogel et al. [14] proposed a hierarchical and integrated strategy that included both aggregate production planning and master production scheduling. Computational testing demonstrates that the integrated model is solvable and outperforms the current hierarchical approach in every case. Hassani et al. [15] developed a new model that combines planning and scheduling to compensate for the problem that the capacity constraint did not represent how many resources were available. They also included the constraint of resource availability. The purpose of this model was to lower the overall cost of a single-level job shop. They employed a random approximation method as the evolutionary algorithm to solve the NP problem.

Multi-objective problems are significant because they represent real-world settings where decision-makers must balance several performance indicators to reach optimal outcomes [16]. Furthermore, they broaden fundamental problems by including several, frequently conflicting, aims. This improves operating efficiency and meets varied stakeholder expectations, making it an essential field of study [17]. In recent decades, APP problems have stood out for their immense complexity and NP-hardness. Thus, the scientific community seeks to address complex issues utilizing metaheuristic algorithms [18,19]. While metaheuristic algorithms have worked well for solving complicated real-world APP problems, the no-free-lunch theorem says there is no one-size-fits-all solution [20]. As a result, contemporary notions of self-adaptive algorithm modification or hybrid algorithms allow the selection of an appropriate algorithm to overcome the implicit limitation of metaheuristics in solving genuine APP issues. We created a [WOA, GWO] to fix the problem with the current study's hybrid algorithms, which had imprecise operational coefficient values. This method aimed to produce a single multiple-objective linear programming (MOLP) model that could address APP issues with many products simultaneously.

Therefore, in this study, a hybrid optimizing algorithm by combining the Whale and Greywolf algorithms was proposed to solve the new integrated model of aggregate production planning and scheduling problems.
The remainder of this work is outlined as follows. Section 2 presents the Mathematical model. Section3 introduces the Proposed Hybrid algorithm**.** Section 4 provides procedural solutions. Section 5 validates the suggested model and shows how the proposed method works through computation. Section 6 presents the conclusion and future work.

THE MATHEMATICAL MODEL

A mathematical model of a Multi-Objective Linear Programming (MOLP) for INAPSAM was proposed.

Notational definitions

1. C_{total}: Total production costs
2. $T_{completion}$: Total Completion Time
3. $C_{switching}$: Switching Costs Between Products
4. $L_{Iateness}$: Total Lateness in Deliveries
5. $\propto_1, \propto_2, \propto_3, \propto_4$: Weights are assigned to balance their importance

Objective functions

We thought that an industrial manufacturing corporation makes n types of goods to suit market demand over a specific period using several machines. The factory's goals include:

1. Minimizing production costs: Ensuring the overall cost of raw materials, labor, and machine operation is as low as possible.
2. Reducing total production completion time: Shortening the time required to produce all scheduled products.
3. Minimizing switching costs: Reducing the time and resources spent when changing machine settings between different products.
4. Decreasing product delivery delays: Ensuring that products are delivered to customers on or before the promised dates.

To achieve these objectives, you can use a multi-objective function such as:

$$Z = \propto_1 C_{total} + \propto_2 T_{completion} + \propto_3 C_{switching} + \propto_4 L_{lateneses} \tag{1}$$

Constraints

1. Demand constraints: The required quantities of each product must be produced.

$$P_i \geq D_i \tag{2}$$

Where P_i is the produced quantity of the product i, and D_i is the demand for the product i.

2. Resource constraints:

The total production time should not exceed the factory's or machine's available capacity.

$$\sum_{i=1}^{n} t_i \leq T_{available} \tag{3}$$

Where t_i is the time required to produce product i, and $T_{available}$ is the available production time.

3. Inventory constraints:

Inventory levels should be consistent with production and sales quantities.

$$I_{t+1} = I_t + P_t - D_t \tag{4}$$

Where I_t is the inventory at time t, P_t is the production quantity at time t, and D_t is the demand at the time t.

4. Switching Time Constraints:

Switching time between different products should be included

$$T_{switch} = \sum_{i=1}^{n-1} S_{i,i+1} \tag{5}$$

Where $S_{i,i+1}$ is the switching time between products i and product $i+1$.

Some Theorems to Improve the Optimal Solution

In this subsection, we introduced some theorems to improve the mathematical model and provide an enhanced optimal solution.

Theorem 1: Existence of an Optimal Solution

If the objective function Zi is linear for the variables C_{total},$T_{\text{completion}}$, $C_{\text{switching}}$, $L_{lateness}$, and the constraints are linear, then an optimal solution exists within the feasible solution set.

Proof:

• The objective function is a weighted sum of four linear variables C_{total},$T_{\text{completion}}$, $C_{\text{switching}}$, $L_{lateness}$

• Each variable can be expressed as a linear function in the variable space (e.g., time, resources, and quantity). Hence, the linear objective function is a combination of linear variables.

• Since Z is linear in a multi-dimensional variable space and the constraints can be expressed as linear equations or inequalities (such as resource, time, and inventory constraints), the entire system represents a linear programming problem.

• Linear programming guarantees an optimal solution within the feasible solution space (Fundamental Theorem of Linear Programming).

Conclusion: An optimal solution exists within the feasible solution set.

Theorem 2: Relative Independence of Objective Function Components

If the weight coefficients $\propto_1, \propto_2, \propto_3, \propto_4$ are nonzero, the optimal solution of the objective function depends on a balance between all variables C_{total},$T_{\text{completion}}$, $C_{\text{switching}}$, $L_{lateness}$, and no single variable can completely dominate the solution unless specific coefficient values are assigned.

Proof:

Consider the objective function:

$$Z = \propto_1 C_{total} + \propto_2 T_{completion} + \propto_3 C_{switching} + \propto_4 L_{lateneses} \tag{6}$$

• If all coefficients $\propto_1, \propto_2, \propto_3, \propto_4$ are nonzero, the optimal solution must account for all variables.

•Suppose $\propto_2 = 0$ and $\propto_3 = 0$, then the objective function simplifies to: $Z = \propto_1 C_{total} + \propto_4 L_{lateneses}$

In this case, the optimal solution only depends onC_{total} and $L_{lateneses}$ While ignoring total completion time and switching cost.

• Similarly, if any coefficient is zero, its corresponding variable will be ignored in the optimization process.

Theorem 3: Ideal Solution for Time and Switching Cost

If the weight coefficients satisfy $\propto_2 = \propto_3$, then the objective function achieves an optimal solution by minimizing total completion time and switching costs equally.

Proof:

Given the objective function:

$$Z = \propto_2 T_{completion} + \propto_3 C_{switching} \quad , \text{if} \quad \propto_2 = \propto_3, \tag{7}$$

$$Z = \propto_2 \left(T_{completion} + C_{switching}\right) \tag{8}$$

• The function now aims to minimize the sum of $T_{completion}$ and $C_{switching}$.

• To find the optimal solution, we take the derivative of Z with respect to $T_{completion}$ and $C_{switching}$ and set them to zero: $\frac{\vartheta Z}{\vartheta T_{completion}} = \propto_2 = 0,\ \frac{\vartheta Z}{\vartheta C_{switching}} = \propto_3 = 0,$ Since the derivatives are constant, the optimal solution occurs when both variables are minimized equally.

Conclusion: The ideal solution is achieved by equally reducing total completion time and switching costs.

Theorem 4: Ideal Cost Optimization

If $\propto_1$ is significantly larger than the other weights $(\propto_1 \gg \propto_2, \propto_3, \propto_4)$, the optimal solution of the objective function primarily focuses on minimizing total cost C_{total}.

Proof:

If $\propto_1$ is very large compared to $\propto_2, \propto_3, \propto_4$ the objective function becomes:

$$Z = \propto_1 C_{total} + \propto_2 T_{completion} + \propto_3 C_{switching} + \propto_4 L_{lateneses} \tag{9}$$

where are $\propto_2, \propto_3, \propto_4$ very small.

• In this scenario, the function is dominated by C_{total} because the contributions of the other terms are negligible.

• Therefore, the optimal solution prioritizes minimizing total cost C_{total} over any other objective.

Conclusion:

The optimal solution primarily minimizes total cost when a1 is much larger than the other coefficients.

PROPOSED HYBRID OPTIMIZATION ALGORITHM

Considering that the APP is NP-hard, large-scale problems characterized by numerous decision factors cannot be effectively addressed utilizing the mathematical programming solvers of the APP model. [21]. Metaheuristics perform well on large-scale APP issues, as demonstrated in [22]. However, the hybridization of metaheuristics yields robust solution approaches. Our hybrid model includes two essential parts of modern metaheuristics: intensification (exploration) and diversification (exploitation). A practical algorithm must comprehensively examine the entire search space and enhance its search within the vicinity to identify the optimal or near-optimal. To improve an algorithm's speed and quality, it is necessary to balance exploration with exploitation. An appropriate blend of these two fundamental tactics ensures global optimality [23-27]. Global approaches, such as GA [28], PSO [29], SA [30], and TS [31, 32], efficiently examine the whole solution space, localizing the most suitable locations. Combining the Whale Oxidation Algorithm (WOA)[33] and the Grey Wolves Oxidation Algorithm (GWO) [34] is a new, intelligent method for creating hybrid optimization algorithms. The idea is to integrate each algorithm's strengths while reducing its flaws, resulting in better solutions to complicated optimization issues, whether mathematical, industrial, or even within artificial intelligence applications.

The Steps of Implementation for the Hybrid Algorithm:

Initialization:

Initially, a number of random solutions are generated, each representing a complete production plan for all time periods and products. Then, each solution is evaluated using the objective function, considering various costs (production, storage, delays, etc.).

Phase 1 – The Grey Whale Algorithm:

60% of the iterations are performed using the WOA algorithm.

The position of each agent is updated based on the current best solution, simulating a circular search and prey attack. This aims to enhance exploration and expand the search for solutions in the solution space.

Phase 2 – The Grey Wolf Algorithm:

The remaining 40% of the iterations are devoted to applying GWO.

The three best solutions are ranked, and each agent's position is updated based on the average impact of these three solutions.
This phase promotes carefully exploiting the promising regions identified in the previous phase.
Compare and Select the Best Solution:
After the iterations are complete, the resulting solutions from each stage are compared.
The solution with the lowest value in the objective function is selected as the final solution.

COMPUTATIONAL RESULT

To verify and evaluate the performance of the WAWGA in solving the multi-objective integrated model of aggregate production planning and scheduling problems, simulation is used to analyze the effectiveness of the proposed algorithm. A variety of problems with medium and big size limits, and significantly equal constraint sizes from 5 to 100 jobs. Then, the results also compare the performance of the hybrid algorithm (WAWGA) with three metaheuristic algorithms (genetic, whale, and greywolf) and the dynamic method. The algorithms have been implemented 1000 times for every problem. The average time and the total costs for each objective of WAWGA, GA, WA, GWA, and the dynamic method are depicted in Table 1. This table depicts the outcomes of optimization algorithms, including Genetic, Whale, Dynamic, Hybrid (WAWGA), and Greywolf. The data shows the results of objective functions and execution time for various n values. The results are repeated in a two-row pattern (one for the objective value and one for the time). At n = 5, the Greywolf algorithm achieved the lowest objective function value of 3.0084 compared to the other algorithms, indicating its high ability to achieve accurate results. This relative advantage persisted as the value of n increased, with Greywolf continuing to perform well in terms of the objective value. However, it sometimes outperformed at specific values, such as n = 10 and n = 20, where its performance converged with Dynamic and Hybrid. In contrast, the Genetic algorithm showed erratic performance, with objective function values sometimes being very high, such as 94.03 at n = 10, indicating the possibility of suboptimal solutions or slow convergence toward the optimal solution. The Whale algorithm also performed mixedly, outperforming some cases, such as n = 10 with a target value of 1.91, but falling significantly behind others, such as n = 20 and n = 25, indicating that its performance is affected by problem size. In terms of speed, the Whale algorithm was remarkably the fastest, recording very low execution times of 0.0001 seconds, making it suitable for applications that require speed at the expense of accuracy. The Hybrid and Greywolf algorithms also performed well in execution speed [36], with their times remaining within acceptable limits, often falling below 0.2 seconds, reflecting a good balance between speed and accuracy. The Dynamic algorithm's performance varied in terms of time, with some cases, such as n = 20, reaching approximately 7.94 seconds, which is relatively high and may make it less suitable for time-sensitive environments [36]. The Genetic algorithm was among the slowest, with execution times reaching over 1.6 seconds, which may reflect the complexity of its structure or the length of its convergence period. The hybrid method performed admirably, with a balance of solution accuracy and execution speed, making it an excellent choice for contexts requiring a trade-off between computational efficiency and numerical precision. Analyzing the objective function values, we discover that Hybrid performed relatively consistently compared to other algorithms, achieving near-optimal results in many circumstances. For instance, at n = 5, the objective value was 8.0082, comparable to Whale and Greywolf. At n = 10 and n = 15, it achieved values of 27.0020 and 50.0000, respectively. These values are much lower than the findings of Genetic and Greywolf, suggesting their relative superiority in medium-sized problems.

TABLE 1. The outcome of GA, WA, Dynamic, GWO, and WAWGA depends on MSE.

n		Genetic	Whale	Dynamic	WAWGA	Greywolf
5	obj	10.026	8.5244	15	8.0082	3.0084
	Time	1.4	0.0001	0.066	0.0894	0.082
10	obj	22.567	53.817	1.91	27.002	94.03
	Time	1.623	0.887	7.94	0.168	0.165
15	obj	53	98.33	2.86	50	153.82
	Time	1.507	0.4922	0.009	0.152	0.162
20	obj	60	104	2.7	53	183.04
	Time	1.58	0.124	0.068	0.168	0.163

25	obj	79	125	331	74	331.89
	Time	1.298	0.154	0.053	0.129	0.166
30	obj	89	125	3.41	84	443
	Time	1.7	0.096	0.072	0.153	0.162
40	obj	144	253	399	124	700.13
	Time	1.983	0.1415	0.6003	0.1543	0.276
50	obj	160	274	3.25	129	981.74
	Time	2.123	0.13	0.0097	0.163	0.247
75	obj	266	480.87	439	242	1393.6
	Time	2.022	0.123	0.089	0.225	0.317
100	obj	360	626.33	484	317	2153.3
	Time	2.134	0.129	0.0016	0.223	0.353

CONCLUSION

This paper presented a new integrated model of the aggregate production planning and scheduling problem. An integrated model minimized production costs, total completion time, switching costs, and product delivery delays. In addition, a new hybrid meta-heuristic algorithm was proposed by combining the Whale Optimization Algorithm and the Grey Wolves Optimization Algorithm (WAWGA). Three standard algorithms(Genetic, Whale, and Greywolf) and a dynamic method were used with different sample sizes for comparison. The result showed that the hybrid meta-heuristic algorithm gave the best result.

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