A Novel Hybrid WOFOA Algorithm for Efficient Multi-Objective APP Solutions

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Abstract. Recently, aggregate production planning has become increasingly important and complex. Therefore, this paper introduces a new multi-objective model for aggregate production planning that aims to minimize production costs, total production time, changeover costs, and product delivery delays. Additionally, a new hybrid metaheuristic algorithm (WAFOA) is proposed, which combines the Whale Optimization Algorithm and the Fly Optimization Algorithm to solve a multi-objective aggregate production planning problem. Three standard metaheuristic algorithms (Genetic, Whale Optimization, and Fly Optimization) are also tested with varying sample sizes for comparison. The results showed that the hybrid metaheuristic algorithm produced the best outcomes.

Keywords: Whale algorithm, Fly Optimization Algorithm, Aggregate Production Planning, multi-objective model, Genetic algorithm

INTRODUCTION

Aggregate Production Planning (APP) problems are important in many manufacturing companies. Managers recognize that personnel and production decisions, which adapt to changing client needs, can have a significant impact on the company's profitability [1]. APP is described as planning production quantities and schedules for a medium-term timeframe of 3 to 18 months [2]. During this stage, the amount of production needed to meet expected demand is figured out. APP tries to establish total production levels to suit future variable or unknown demand in each product category [3]. It also considers policy and decision-making factors such as hiring, overtime, layoffs, backorders, subcontracting, and inventory management [4]. Since the 1950s, different APP models with varying levels of complexity have been developed. As noted by [5], traditional methods for addressing these problems can be categorized into linear decision rules [6], linear programming [7-9], transportation methods [10], management coefficient approaches [11], search decision rules [12], and simulation [13]. In recent decades, APP problems have become highly complex and NP-hard. Thus, the researchers have focused on solving these complicated problems using metaheuristic algorithms [14-20]. Although metaheuristic algorithms have effectively been used to address complicated real-world APP problems, no single solution is effective for all situations due to the no-free lunch theorem [21]. So, modern ideas like self-adaptive modification of algorithms or hybrid algorithms that help you choose the right method try to get around the hidden challenges that metaheuristics have when they try to solve real APP problems . Several researchers have utilized metaheuristic algorithms, such as a hybrid model of genetic algorithm (GA) and ant colony optimization, to solve the APP of long-term policies in industries [22]. Additionally, a mixed-integer linear programming model was created for a generalized two-phase APP method [23]. After that, the genetic algorithm and tabu search methods were used to work on the APP model. In [24], To solve the integer-based linear programming model for APP problem sets, updated particle swarm optimization (PSO) strategies were suggested. Also, the assumption is that imprecise deterministic parametric values can give outcomes that are neither useful or practical [25- 28]. Most models for solving APP problems were focused on individual objectives and are incompatible with actual production planning systems. As well as, these methods mainly concentrate on solution algorithms without considering comprehensive, generalized models, making them incompatible with real production systems. As a result, no generalized and comprehensive models have been developed to adapt to actual production environments. The present study proposed a new hybrid algorithm (WOFOA) for solving the multi-objective APP problem. The remainder of this work is organized as follows: the mathematical programming model is presented first, followed by the proposed Hybrid Algorithm. Next, the computational study and results are discussed, and finally, the conclusion along with potential directions for future work are provided.

THE MATHEMATICAL MODEL

A mathematical model of a Multi-Objective Linear Programming (MOLP) for INAPSAM was proposed.

Notational definitions

w_t → Regular time labor cost per hour at each period t.
M_t → labor hours of regular time at each period t.
W_t → Work force sizes of worker type j at period t.
o_t → Over time labor cost per hour at each period t.
O_t → Namber of over time hours of worker at period t.
h_t → Hiring and training cost per worker of period t.
H_t → Workforce hired at the beginning of period t.
f_t → Layoff cost per worker of period t.
F_t → Workforce laid off at beginning of period t.
i_{nt} → Inventory holding cost per unit of product at each period t.
I_{nt} → Units of inventory of product at the end of period t.
b_{nt} → Backlog cost per unit of product at each period t. ,
c_{nt} → Unit material cost.
p_{nt} → Units of product produced at period t.
s_{nt} → Subcontracting cost per unit of product.
S_{nt} → Units of product subcontracted at period t.

Objective functions

$$Min\ D = \sum_{t=1}^{T}\sum_{n=1}^{N} w_t M_n W_t + \sum_{t=1}^{T} o_t O_t + \sum_{t}^{T}(h_t H_t + f_t F_t) + \sum_{n=1}^{N}\sum_{t=1}^{T}(b_{nt}B_{nt} + i_{nt}I_{nt}) + \sum_{n=1}^{N}\sum_{t=1}^{T}(c_{nt}p_{nt} + s_{nt}S_{nt}) \quad (1)$$

Constraints

Inventory level constraint:

$$p_{nt} + I_{n(t-1)} - I_{nt} + S_{nt} - B_{n(t-1)} + B_{nt} = D_{nt}, \quad \forall n, \forall t. \quad (2)$$

Capacity constraint:

$$\sum_{n=1}^{N}(AR \times P_{nt}) - (M_n \times W_t) - O_t \leq 0 \ , \forall t. \quad (3)$$

Workforce, hiring, and layoff constraints:

$$F_t - H_t + W_t - W_{t-1} = 0 \quad \forall t. \quad (4)$$

Non-negativity constraints:

$$P_{nt}, I_{nt}, B_{nt}, S_{nt}, O_t, H_t, F_t, W_t \geq 0 \qquad \forall n, \forall t\ .5.4.2.1 \quad (5)$$

PROPOSED HYBRID ALGORITHM

Hybrid metaheuristic algorithms are effective solution techniques. They are created by merging two or more algorithms to increase the total search efficiency. A practical algorithm must exhaustively explore the entire search space and refine its search locally to find the optimal or near-optimal solution [29-35]. Accordingly, this study proposes a hybrid algorithm that balances exploration and exploitation to improve the speed and quality of the search by integrating the Fly Optimization Algorithm with the whale algorithm. The steps of the hybrid algorithm are outlined as follows:

The Steps of Implementation for the Hybrid Algorithm:

Step 1. Initialization

Initialize the population size n, the number of decision variables d, and the limits of the search space[lb, ub].

Generate an initial population Xi $\in R^d$, i=1,2,..., n uniformly at random within the search space.

Calculate the fitness of all members concerning the objective function.

Find the best solution X^* based on the minimum objective value.

Step 2. Whale Optimization Phase (Global Search)

It is run for almost 60% of the total number of iterations. It is based on humpback whales' bubble-net hunting strategy, and comprises two main behaviors:

- Encircling the Prey: Updating the position of each whale by:

$$\overrightarrow{A}(t+1) = \vec{X}^* - \overrightarrow{A}\,.\,|\overrightarrow{C}\,.\,\vec{X}^* - \vec{X}^*(t)|$$

Where $\overrightarrow{A} = 2a.r_1 - a$, $\overrightarrow{C} = 2.r_2$, a decrease linearly from 2 to 0 over iterations, r_1 , r_2 $\in$[0,1] are random vectors.

- Spiral Updating: Updating the position with 50% probability using a spiral equation:

$$\overrightarrow{X}(t+1) = |\vec{X}^* - \overrightarrow{X}(t)|.\,e^{bl}\,.\cos(2\pi l)|\overrightarrow{C} + \vec{X}^*$$

$l \in [-1,1]$ is a random number.

Step 3. Fly Optimization Phase (Local Search)

After the WOA phase, FOA is executed for 40% of all iterations to enhance the best solution. FOA generates new candidate solutions around the current best solution using: $\overrightarrow{X}_{new} = \vec{X}^* + \Delta$

Δ is a small perturbation vector (usually Gaussian or uniform noise).

The fitness of all generated solutions is evaluated.

The best one among them replaces $\vec{X}^*$ if it improves the objective value.

Step 4. Selection and Comparison

Compare the best solutions obtained from both WOA and FOA phases.

The overall best solution X_{hybrid} = arg min{f(X_{WOA}),f(X_{FOA})}

Step 5: Local Fine-Tuning (Post-Optimization Adjustment)

Apply a local perturbation procedure for a fixed number of iterations about the hybrid solution:

$$\overrightarrow{X}_{trial} = \overrightarrow{X}_{hybrid} + N(0,\sigma)$$

If any perturbed solution improves the objective function, it is adopted as the new best.

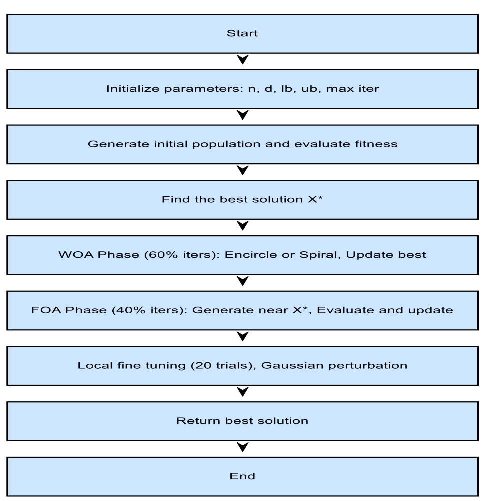

FIGURE 1. Flowchart of the WOFOA

COMPUTATIONAL STUDY AND RESULT

To verify and evaluate the performance of the WOFOA in solving the multi-objective aggregate production planning, simulation is used to analyze the effectiveness of the proposed algorithm. A variety of problems with medium and large sizes (10, 20, 30,40,75, 100, 150, 200, 300, 500, and 700 jobs). Then, the results also compare the performance of the hybrid algorithm (WOFOA) with three metaheuristic algorithms (genetic, whale, and Fly Optimization). The algorithms have been implemented 1000 times for every problem. The average time and the total costs for each objective of WOFOA, GA, WA, and FOA are depicted in Table 1. This table describes the outcomes of optimization algorithms, including Genetic, Whale, Hybrid (WOFOA), and Fly Optimization. The data shows the results of objective functions and execution time for various n values. This evaluation focused on two main criteria: solution quality (obj value) and execution efficiency (time). Regarding solution quality, the hybrid algorithm achieved the lowest obj values at all data sizes, indicating its strong ability to approach, and in some cases surpass, optimal solutions. For instance, at n = 150, the hybrid algorithm recorded an objective function value of approximately 2216.8, compared to: GA = 16023, WOA = 17533, FOA = 13802. At n = 750, the hybrid further improved its performance to achieve 11142, while the other algorithms recorded: GA = 1.1582e+, WOA = 86498, FOA = 1.1094e+. These significant differences highlight the hybrid algorithm's capability to find better and more stable solutions as data size and problem complexity increase.

TABLE 1. The outcome of GA, WA, FOA, and WAFOA depend on MSE.

n		GA	WOA	Fly	Hybrid
10	obj	792	1106.9	711.6	97.219
	Time	0.0768	0.0338	0.029	0.0221
20	obj	1230	1798.1	1127.1	165.11
	Time	0.0029	0.0192	0.0266	0.0238
30	obj	2286	3350.9	2002.2	413.39
	Time	0.0901	0.0179	0.0192	0.0118
40	obj	2924.2	4258.8	2712.6	426.48
	Time	0.0488	0.0177	0.0162	0.0105
75	obj	6446.8	7960.8	6105.4	1033.1
	Time	0.9673	0.0187	0.0225	0.0118
100	obj	9757.3	11097	7669.6	1508.6
	Time	0.0602	0.0215	0.0239	0.0135
150	obj	16023	17533	13802	2216.8
	Time	0.0958	0.0509	0.0448	0.0336
200	obj	23460	22303	20438	2829
	Time	0.1192	0.0236	0.0461	0.0264
300	obj	40871	35486	36026	4535.4
	Time	0.0923	0.0324	0.0441	0.0264
500	obj	72065	55783	65222	7259.6
	Time	0.1527	0.0639	0.0857	0.0371
750	obj	1.1582e	86498	1.1094e	11142
	Time	0.1609	0.0535	0.0762	0.0405

In terms of execution time, the hybrid algorithm maintained excellent performance, often ranking as the fastest or second-fastest algorithm. For example: At n = 10, Hybrid was faster at 0.0221 seconds compared to GA (0.0768), WOA (0,0338), and Fly (0.029). At n = 750, Hybrid achieved an execution time of only 0.0405 seconds, while GA took 0.1609 seconds, WOA took 0.0530 seconds, and Fly took 0.0762 seconds. This excellent balance between accuracy and computational speed highlights the competitive advantage of the hybrid algorithm, particularly in environments that require both fast and high-quality solutions. It is also noted that the hybrid algorithm's performance remained stable as data volume increased, whereas the other algorithms' performance deteriorated, either with high execution times or lower solution quality. These results demonstrate that the hybrid algorithm combines high computational efficiency with superior solution quality, outperforming other traditional algorithms, particularly for large-scale and highly complex problems. Therefore, adopting the hybrid algorithm is a strategic choice for any applied optimization system in artificial intelligence, optimization, or big data analysis fields, where processing demands accurate solutions with rapid response times.

CONCLUSION AND FUTURE WORK

This paper introduced a new multi-objective model for the aggregate production planning problem. The model aimed to minimize production costs, total completion time, switching costs, and product delivery delays. Additionally, a new hybrid metaheuristic algorithm was proposed by combining the Whale Optimization Algorithm and the Fly Optimization Algorithm (WOFOA). Three standard algorithms GA, WOA, and Fly were also included. The results showed that the hybrid metaheuristic algorithm produced the best outcome.

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