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## Reduced Transition Probabilities for Even-Even Nuclides (Po and Rn) with A=206-222

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# Reduced Transition Probabilities for even-even nuclides (Po and Rn) with A=206-222

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**Abstract:** We computed the electric quadrupole transition and determined the relationship between the number of neutrons and  $M(E2)$  [2w.u.] for gamma rays from 2+ to 0+ using Ferston's half-life time for the 84Po and 86Rn isotopes for even-even (A=206- 222). We determine the empirical formula for these relations using the MATLAB tool. For the 84Po and 86Rn nuclides enumerated and displayed, the computed decreased transition probabilities  $B(E2) e^2 b^2 \uparrow$  values are compared with theoretical and experimental predictions, and they show good agreement with SSANM and FRDM as well as with experimental values of Global. When determining the present work of the transition probability (T), the mean life time ( $\tau$  (s)), and the theoretical value of the energy state (first and ground) of a strontium isotope, By computing the theoretical value of the total width for gamma decay and the Weisskopf  $\Gamma(E2) w. u$  energy (the Weisskopf single-particle widths), the theoretical value of the energy state (first and ground) and the present action of the transmission force  $|M(E2)|^2 W. u \downarrow$ , This is part of the formula where A is the mass number and  $E_\gamma$  is computed in keV. After tabulating, discussing, and drawing the results, it was discovered that the value of  $|M(E2)|^2 W. u \downarrow$  is at its lowest when the magic number of neutrons is equal to 128 and that the values rise as we move away from the magic numbers, that is, when the nuclei are saturated for even-even nuclei and the same subject, regardless of the number of protons.

**Keywords:** Isotopes of radon, polonium, electric quadrupole transition, and decreased transition probabilities.

## INTRODUCTION

Comprised of interacting nucleons held together by the strong nuclear force, the atomic nucleus is a complicated quantum system. The short-range nature of the strong interaction and the nucleus's limited spatial extent allow nuclei to display relatively simple excitation patterns. State lifetimes can range from extremely short intervals (on the order of  $10^{-24}$  seconds) to much longer timescales, such as hours or even years. Finding electromagnetic transition probabilities is a crucial step in examining the structure and underlying dynamics of these excited states. This is accomplished by combining the examination of internal conversion coefficients,  $\gamma$ -ray intensities, transition multipolarities, and observed level lifetimes. [1].

An essential part of studying nuclear structure is figuring out nuclear level lifetimes and related transition probabilities. Decreased transition probabilities provide important insights into the internal structure of nuclei, especially the characteristics of excited states, including their electromagnetic matrix elements and wave functions. [2]. A wide range of experimental methods have been developed over the years to measure lifetimes, including Coulomb Excitation, Electronic Timing, and Recoil Distance Doppler Shift (RDDS). These methods are based on fast-timing techniques that use  $\gamma$ -ray and/or particle coincidence measurements for direct lifetime determination [3,4]. These methods now span a temporal range from the femtosecond to the microsecond scale. The processes used to extract transition probabilities from recorded lifetimes have remained rather consistent throughout the field, regardless of the particular approach taken. [2]. A reduced transition probability can be obtained by comparing the experimental result with the theoretical lifetime of the  $\gamma$ -ray transition. [5], i.e.

$$\text{Reduced transition probability} = \frac{T_{th}}{T_{exp}} \quad (1)$$

$$T = \frac{1}{\tau} \quad (2)$$

where the transition probability is denoted by T. The relationship between the mean lifetime  $\tau$  and the half-life time  $t_{1/2}$  is as follows: [6]:

$$t_{1/2} = T \ln 2 \quad (3)$$

$\tau = t_{1/2} \ln(2)$  where  $\tau$  is the mean lifetime.

Ferston [7] has been used to compute the electric quadrupole transition strengths  $|M(E2)|_{2w.u.}$  for gamma rays from 2+ to 0+ to obtain the half-life for the first excited state as follows:

The total width for gamma decay is given by [8]:

$$\Gamma_\gamma = \sum \Gamma_\Pi \quad (4)$$

$$\text{Where } \Gamma_\gamma \tau \approx \hbar = 0.658212 \times 10^{-15} \text{ eV.S} \quad (5)$$

The gamma ray transition strength  $|M(E2)|^2$  is defined as [9]

$$|M(E2)|^2 = \frac{\Gamma_\gamma}{\Gamma(E2)_{w.u.}} \quad (6)$$

Where  $\Gamma(E2)_{w.u.}$  the Weisskopf single-particle widths

$$\Gamma(E2)_{w.u.} = 4.7907 \times 10^{-23} A^{4/3} E_\gamma^5 \quad (7)$$

Where  $E_\gamma$  is calculated in keV and A is the mass number

Gamma-rays are electromagnetic radiation that frequently occurs during an isomeric transition from the nucleus's upper energy state to its lower energy level through emission. A single 2L-pole quantum can be emitted during the  $\gamma$ -transition from an initial state of total angular momentum  $J_i$  and parity  $\pi_i$  [9] to a final state of total angular momentum  $J_f$  and parity  $\pi_f$ . [9]

$$|J_i - J_f| \leq L \leq J_i + J_f \quad \text{for } L \neq 0 \quad (8)$$

Where L, which is a multipolarity, is the angular momentum of the  $\gamma$ -transition [10]. The parity change of electric radiation (EL) in such a transition is provided by

$$\pi_i \pi_f = (-1)^L \quad (9)$$

The relation between the reduced transition probabilities,  $B(EL)_{\downarrow} = B(EL, 2 \rightarrow 1)$  and  $B(EL)_{\uparrow} = B(EL, 1 \rightarrow 2)$ , is given by [11]:

$$B(EL)_{\downarrow} = \frac{2J_i + 1}{2J_f + 1} B(EL)_{\uparrow} \quad (10)$$

The best theoretical models to calculate  $B(E2)_{\uparrow}$  are Nilsson Single-Shell Asymptotic One of the most straightforward theoretical models for comprehending  $B(E2)_{\uparrow}$  is Model SSANM, which is predicated on the idea that the nucleus is as distorted as possible within a single shell. This model has been thoroughly examined in reference [12], where the  $B(E2)_{\uparrow}$  values in  $e^2 b^2$  units are provided by:

$$B(E2)_{\uparrow} = \frac{5}{16\pi} [e^2 Q_0]^2 \quad (Q_0 \neq 0) \quad (11)$$

Where  $Q_0$  the intrinsic quadrupole momentum [2].

The nuclear ground state forms in the Finite-Range Droplet Model (FRDM) [6] are determined by minimizing the nuclear potential energy function with respect to the  $\epsilon_2$ ,  $\epsilon_3$ ,  $\epsilon_4$ , and  $\epsilon_6$  shape degrees of freedom. See ref. [13] for further information on this model. Basic experimental quantities that are independent of nuclear models are the  $B(E2)$  values. The value of this (Weisskopf) single-particle  $B(E2)_{\uparrow}$  is provided by [10].

$$B(E2)_{\uparrow} = 2.6 E^{-1} Z^2 A^{-2/3} \quad (12)$$

## RESULTS AND DISCUSSION

Recent research has been conducted to evaluate the characteristics of even-even ( $^{84}\text{Po}$  and  $^{86}\text{Rn}$ ) nuclei by studying the electric quadrupole transitions ( $E2: [2]_{-1}^{+} \rightarrow 0_{-1}^{+}$ ). We computed the electric quadrupole transition strengths  $|M(E2)|_{2w.u.}$  for  $\gamma_0$ -transition as a function of neutron number (N) for even nuclei of isotopic (120-136) for ( $^{84}\text{Po}$ - $^{86}\text{Rn}$ ) using half-life ( $t_{1/2}$ ), energy of the first excited state, and  $\gamma_0$ -energy from Ferston [7]. The results of calculations of mean life ( $\tau$ ), the total width for gamma decay ( $\Gamma_\gamma$ ), gamma Weisskopf ( $\Gamma_{w.u.}$ ), and  $|M(E2)|_{2w.u.}$  are presented for all even nuclei listed in table. (1)(2).

Figures 1 and 2 show that the relationship between  $|M(E2)|_{2w.u.}$  and the number of neutrons is greatest at the magic number 128. However, for values less and greater than 128, the  $|M(E2)|_{2w.u.}$  values are lower for the isotopes of the elements  $^{84}\text{Po}$  and  $^{86}\text{Rn}$ , respectively.

**TABLE 1.** Transition strengths  $|M(E2)|_{2w.u.}$  of  $\gamma_0$ -transition from  $2^+ \rightarrow 0^+$  with partial gamma widths in W.u.  $\Gamma(E2)_{w.u.}$ , total gamma width and mean lifetime  $\tau$  for the first excited state of  $^{84}\text{Po}$  isotope (present work)

A	N	$E_i(\text{keV})$ [7]	$E_\gamma(\text{keV})$ [7]	$t_{1/2}$ [7]	T(s) P.Work	$\Gamma_{\text{tot}}(\text{eV})$ P.Work	$\Gamma(E2)_{w.u.}(\text{eV})$ P.Work	$ M(E2) _{2w.u.}$ P.Work
206	122	700.66	700.66	8.8d	1.0971e+06	5.9993e-22	9.8422e-06	6.0955e-17
208	124	686.528	686.527	2.898y	1.3007e+08	5.0604e-24	9.0041e-06	5.6201e-19
210	126	1181.40	1181.39	138.376d	1.7252e+07	3.8153e-23	1.3762e-04	2.7724e-19
212	128	727.330	727.330	0.299 $\mu\text{s}$	4.3146e-07	1.5256e-09	1.2326e-05	1.2376e-04
214	130	609.316	609.312	164.3 $\mu\text{s}$	2.3709e-04	2.7763e-12	5.1503e-06	5.3905e-07
216	132	549.76	549.76	0.145s	0.2092	3.1458e-15	3.1180e-06	1.0089e-09

**TABLE 2.** Transition strengths  $|M(E2)|_{2w.u.}$  of  $\gamma_0$ -transition from  $2^+ \rightarrow 0^+$  with partial gamma widths in W. u.  $\Gamma(E2)_{w.u.}$ , total gamma width and mean life time  $\tau$  for the first excited state of  $^{86}\text{Rn}$  isotope (present work)

A	N	$E_i(\text{keV})$ [7]	$E_\gamma(\text{keV})$ [7]	$t_{1/2}$ [7]	T(s) P. Work	$\Gamma_{\text{tot}}(\text{eV})$ P. Work	$\Gamma(E2)_{w.u.}(\text{eV})$ P. Work	$ M(E2) _{2w.u.}$ P. Work
206	120	575.3	575.31	5.87m	508.2251	1.2951e-18	3.6731e-06	3.5260e-13
208	122	635.8	635.82	24.35m	2.1082e+03	3.1221e-19	6.1341e-06	5.0897e-14
210	124	643.8	643.81	2.4h	1.2468e+04	5.2794e-20	6.6137e-06	7.9825e-15
212	126	1273.8	1273.82	23.9m	2.0693e+03	3.1809e-19	2.0309e-04	1.5663e-15
214	128	694.7	694.7	0.27 $\mu\text{s}$	3.8961e-07	1.6894e-09	9.9221e-06	1.7027e-04
216	130	461.9	461.9	45 $\mu\text{s}$	6.4935e-05	1.0136e-11	1.3054e-06	7.7650e-06
218	132	324.22	324.22	35ms	0.0505	1.3033e-14	2.2519e-07	5.7875e-08
220	134	240.986	240.986	55.6s	80.2309	8.2040e-18	5.1712e-08	1.5865e-10
222	136	186.24	186.24	3.8235d	4.7670e+05	1.3808e-21	1.4429e-08	9.5695e-14

For ( $^{84}\text{Po}$ - $^{86}\text{Rn}$ ) with just one transition for gamma ray ( $\gamma$ ) and ( $\gamma_0$ ) with intensity (100%)  $E2$ , the  $|M(E2)|_{2w.u.}$  for the  $2^+ \rightarrow 0^+$  transition as a function of neutron number (n) is computed ref.[9] to even – even isotopes. Tables (1) and (2) present the results of the computations for the ( $^{84}\text{Po}$ ) and  $^{86}\text{Rn}$ ) nuclides, respectively. Figures (1) and (2) exhibit the results of  $|M(E2)|_{2w.u.}$  as a function of neutron number (n), while  $B(E2) e^2 b^2$  Equation (12) was used to calculate the findings, which are presented in tables (3) and (4), respectively, as seen in figures (3) and (4). These values are compared with the experimental value and with those in figures (5 and 6).

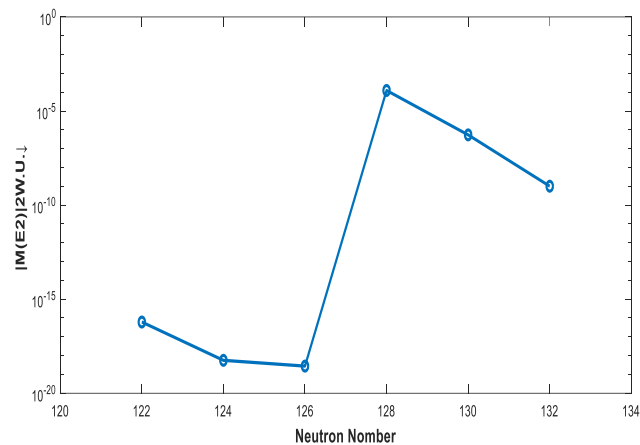
**TABLE 3.** The calculated reduced transition probabilities  $B(E2) e^2 b^2$  values are compared with that of experimental and theoretical predications for  $^{84}\text{Po}$  nuclide

A	N	$E_i(\text{keV})$ [7]	$E_\gamma(\text{keV})$ [7]	$B(E2; 0_1^+ \rightarrow 2_1^+) e^2 b^2$			
				Experimental values of Global [10]	Theoretical values		
					Present work	SSANM [10]	FRDM [10]
206	122	700.66	700.66	0.74 13	0.7507	0.740	0.032
208	124	686.528	686.527	0.75 13	0.7612	0.493	0.032

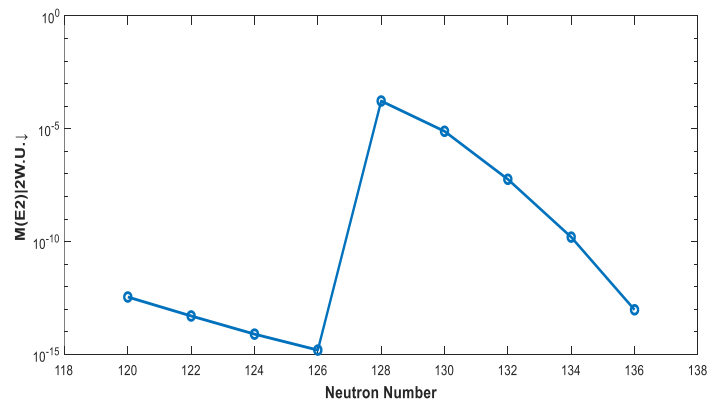
210	126	1181.40	1181.39	0.43 8	0.4395	0.260	-----
212	128	727.330	727.330	0.70 12	0.7094	0.779	-----
214	130	609.316	609.312	0.83 14	0.8416	1.311	0.007
216	132	549.76	549.76	0.91 16	0.9269	1.955	0.046

**TABLE 4.** The calculated reduced transition probabilities  $B(E2) e^2 b^2 \uparrow$  values are compared with that of experimental and theoretical predications for  $^{86}\text{Rn}$  nuclide

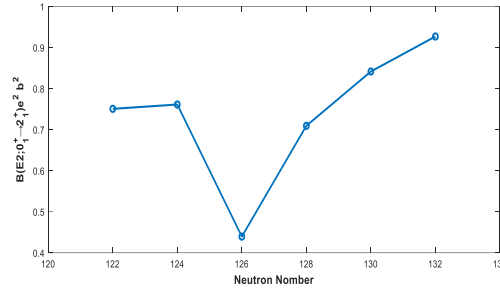
A	N	$E_i(\text{keV})$ [7]	$E_f(\text{keV})$ [7]	$B(E2; 0_1^+ \rightarrow 2_1^+) e^2 b^2$			
				Experimental values of Global [10]	Theoretical values		
					Present work	SSANM [10]	FRDM [10]
206	120	575.3	575.31	0.95 16	0.9583	1.855	0.197
208	122	635.8	635.82	0.85 15	0.8615	1.440	0.070
210	124	643.8	643.81	0.83 15	0.8454	1.081	0.071
212	126	1273.8	1273.82	0.42 7	0.4246	0.713	-----
214	128	694.7	694.7	0.76 13	0.7737	1.496	0.007
216	130	461.9	461.9	1.14 20	1.1564	2.222	0.007
218	132	324.22	324.22	1.62 28	1.6374	3.057	0.202
220	134	240.986	240.986	2.16 38	2.1896	3.621	1.851
222	136	186.24	186.24	2.78 48	2.8162	4.169	3.019



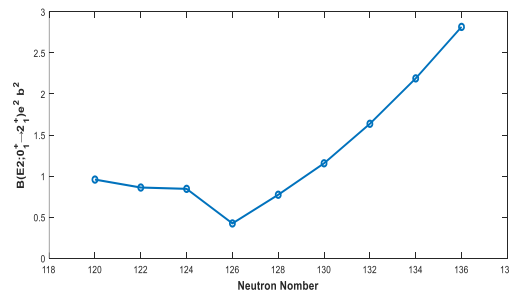
**FIGURE 1.** Relation between the neutron number and the electric quadrupole transition (E2) in  $^{84}\text{Po}$  nuclide



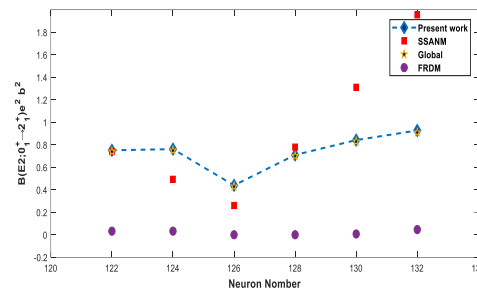
**FIGURE 2.** Relation between the neutron number and the electric quadrupole transition (E2) in  $^{86}\text{Rn}$  nuclide



**FIGURE 3.** The relation between  $B(E2) \uparrow$  values of the present work with mass Number for  $^{84}\text{Po}$

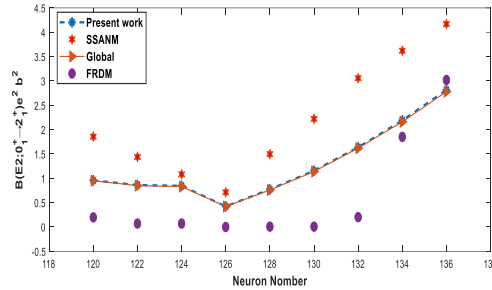


**FIGURE 4.** The relation between  $B(E2) \uparrow$  values of the present work with mass Number for  $^{86}\text{Rn}$



**FIGURE 5.** The Comparison between  $B(E2) \uparrow$  values of the present work for  $^{84}\text{Po}$  nuclei with of ref. [6] experimental and other theoretical results

The relationship between the reduced transition probabilities,  $B(EL)\downarrow$  and the number of neutrons for the isotopes of the elements Po and Rn is at its lowest value at the magic number 128, as shown in Figures 3 and 4, respectively. The current  $B(E2) e2b2\uparrow$  curves correspond well with those of ref. [10], Although for ( $^{84}\text{Po}$ - $^{86}\text{Rn}$ ) nuclides, the experimental and current work values accord more with SSANM than FRDM. Global data for Po120 and  $^{84}\text{Po}132$  were in close agreement with the value of  $B(E2) e2b2\uparrow$  for  $^{84}\text{Po}$ , and global data for Rn136 were in close agreement with the value of  $B(E2) e2b2\uparrow$  for  $^{86}\text{Rn}122$ .



**FIGURE 6.** The Comparison between  $B(E2) \uparrow$  values of the present work for  $^{86}\text{Rn}$  nuclei with experimental and other theoretical results

## CONCLUSION

To explore nuclear structural properties, the electric quadrupole transitions ( $E2: 2_1^+ \rightarrow 0_1^+$ ) in even-even nuclei of  $^{84}\text{Po}$  and  $^{86}\text{Rn}$  were examined in this work. Ferston's [7]  $\gamma_0$ -energy data, half-life ( $t_{1/2}$ ), and energy of the initial excited state were used to compute the electric quadrupole transition intensities for the  $\gamma_0$  transition as a function of neutron number ( $N$ ) for even isotopes with mass numbers ranging from 120 to 136. Each isotope has a single  $E2$  transition with 100% intensity, according to the values for mean life ( $\tau$ ), total gamma decay width ( $\Gamma_\gamma$ ), gamma Weisskopf estimations ( $\Gamma_w.u.$ ), and Figures. This makes analysis easier and improves reliability. Equation (12) was also used to get the reduced transition probabilities in units of  $e^2b^2$ . Figures (5) and (6) show the computed values graphically and compare them to theoretical models and experimental data. The current findings were in good accord with the figures in Ref. [10], especially for both  $^{84}\text{Po}$  and  $^{86}\text{Rn}$  nuclides, matching the SSANM model more closely than the FRDM predictions. In particular, it was discovered that the value of  $^{84}\text{Po}$  nearly matched the global average for the isotopes  $\text{Po}120$  and  $\text{Po}132$ , and that the value of  $^{86}\text{Rn}122$  roughly matched that of  $\text{Rn}136$ . These results illustrate the applicability of the proposed models in characterizing  $E2$  transitions in this region of the nuclear chart and validate the agreement between current theoretical calculations and experimental trends.

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