Study The Effect of Neutron Numbers on The Reduced Transition Probability and Deformation Parameter for Even-Even $_{56}$Ba Isotopes

Murtadha S. Nayyef[1, a)], Naz T. Jarallaha[2,b)], Amal J. Hatem[2,c)]

[1]*Department of Science, College of Basic Education, Mustansiriyah University, Baghdad, Iraq.*
[2]*Department of Physics, College of Education for Pure Sciences/ Ibn AL-Haitham, University of Baghdad, Baghdad-Iraq.*

[a)] *Corresponding author: murtadha199@uomustansiriyah.edu.iq*
[b)] *naz.t.ja@ihcoedu.uobaghdad.edu.iq*
[c)] *amal.j.h@ihcoedu.uobaghdad.edu.iq*

Abstract. Neutron numbers play a crucial role in determining both the reduced electric quadrupole transition probability, B(E2), and the overall geometric shape of the nucleus. In the present study, the evolution of nuclear shape with varying neutron numbers has been investigated for the even–even ^{56}Ba isotopes, spanning the neutron range N = 62–92. To examine this dependence, the calculated values of B(E2) and the corresponding quadrupole deformation parameter, β_2, have been systematically analyzed. Both quantities—B(E2) and β_2—were plotted as functions of the neutron number in order to visualize and assess how increasing or decreasing neutron content influences the degree of collectivity and deformation. The observed trends reveal that the nucleus tends to become more spherical as the neutron number approaches the magic neutron number, reflecting the stabilizing effect of shell closures and the corresponding reduction in deformation and collective motion.

Keywords: Magic number, Reduced transition probability, Nuclear deformation.

INTRODUCTION

At first, it was supposed that the shapes of nuclei were spherical. Later, Wolfgang Pauli proposed that there were other shapes for nuclei when the nucleus was excited. Then, Bohr and Fritz Kalkar found that the shapes of nuclei could be experimentally probed through the measurement of gamma-ray photons emitted during nuclear de-excitation [1]. Nuclei deviate from the spherical shape, undergoing deformation, when the number of constituent nucleons (protons Z and neutrons N) not equal to any of the magic numbers (2, 8, 20, 28, 50, 82, and 126). That is, the nuclei will be more abundant and stable if the number of nucleons (neutrons N and protons Z) in them equals one of the magic numbers [2]. The underlying cause of nuclear deformation stems from the collective ordering of valence nucleons in incompletely filled major shells. Consequently, deformation is primarily observed in nuclei where both the neutron and proton levels are partially filled [3]. The easiest and most common type of non-spherical distortion is the quadrupole deformation; in this case of deformation, the shape of the nuclei is usually either elongation or flattening [4]. The quadrupole deformation parameter is a fundamental quantity for characterizing and quantifying these nuclear shape transitions [5]. For even-even nuclei, moving from the first excited level, typically the 2^+ state, to the 0^+ ground state is of particular importance [6]. Electromagnetic transitions, specifically the electric quadrupole (E2) transitions, serve as a vital source of information for studying nuclear structure [7]. (B(E2; $0^+\rightarrow2^+$)) incorporates crucial nuclear structure information, including the energy of low-lying collective levels. The values of B(E2; $0^+\rightarrow2^+$) are a direct indicator of quadruple distortion in nuclides [8]. Many studies have focused on the form of the nuclide & examined the degree of nuclear distortion [9-13]. In this work, the effect of the number of neutrons on the shape of the nucleus and the degree of distortion in its shape was studied. The deformation parameter (β_2) for the $_{56}$Ba isotopes, which have neutron numbers (62-92), was calculated. To obtain this coefficient, we need to calculate the probability of an electrical transition B (E2; $0^+ \rightarrow 2^+$) from the Global Best fit equation. Finally, our results are compared with theoretical data.

MATERIALS AND METHODS

Reduced Transition Probability B(E2)↑

B(E2) is a fundamental quantity that can be calculated theoretically or derived from experimental data, serving as a basis for comparison with experiment [14]. The nuclear model does not affect on these fundamental experimental quantities. To calculate the B(E2) value, based on Global Best Fit (GLOBAL) we need only to know the energy of the gamma rays for the 2^+ level [15].

$$B(E2)\uparrow = \frac{2.6\,Z^2}{E_{\gamma_0}\,A^{\frac{2}{3}}} \tag{1}$$

Where: E_{γ_0} :is the ɣ energy of transition in (keV) units.

z: proton numbers

A: proton and neutron numbers of a nucleus

Quadruple Deformation Parameter (β_2)

The (β_2) quantifies the extent of the deflection of the nuclear shape from circular symmetry, representing the degree of nuclear quadrupole deformation (Elongation or flattening), which is calculated from the equation below [16,17].

$$\beta_2 = \frac{4\pi}{3ZR_o^2}\left[\frac{B(E2)\uparrow e^2 b^2}{e^2}\right]^{1/2} \tag{2}$$

R_0 is the average nuclear radius calculated by using the following equation

$$R_o^2 = 0.0144\ A^{2/3}\ \text{barn} \quad . (3)$$

METHODLOGY

In the present study, both the transition probability B(E2)↑ for the ($0^+ \rightarrow 2^+$) transition and the deformation coefficient (β_2) were calculated. To apply equation 2, the average radius of the nucleus R_o^2 calculate from equation 3. These parameters were calculated for even-even $_{56}$Ba isotopes which have neutron numbers (62-92). The study also addressed the effect of the number of neutrons on both B(E2)↑ and (β_2) through a graph that illustrates the effect of this relationship between them.

Finally, the relationship between B(E2)↑ and (β_2) of the nucleus was clarified by drawing a graph of (β_2) as a function of B(E2)↑.

RESULTS AND DISCUSSION

The calculated B(E2)↑ transition probabilities for the even–even ^{56}Ba isotopes are presented in Table 1. These calculated values are compared with the corresponding theoretical predictions, which are listed side-by-side in the same table to facilitate direct evaluation of the agreement between experiment and model.
Similarly, the calculated values of the mean-square nuclear radius R_0^2 and the quadrupole deformation parameter β_2 for the even–even ^{56}Ba isotopic chain are presented in Table 2. The computed β_2 values are compared with the available theoretical estimates, which are provided in column 3 of the table, allowing for a clear assessment of deviations and trends across the isotopes.
To better illustrate the nuclear-structure evolution along the isotopic chain, the values of B(E2)↑ and β_2 are plotted as a function of the neutron number (N) in Figures 1 and 2, respectively. These graphical representations highlight the dependence of collective behavior on neutron content.

In Figure 3, the deformation parameter β_2 is plotted as a function of B(E2)↑, providing a direct visualization of the correlation between electric quadrupole transition strength and nuclear deformation.

TABLE 1. Atomic Number A, Neutron Numbers N, Gamma Energy (E_{γ_0}), Transition Probabilities B(E2) e^2b^2↑ for $_{56}$Ba Isotopes.

A	N	Theoretical Values		Present Work
		E_{γ_0} (KeV) [18]	B(E2) ↑e^2b^2 Global Best Fit [15]	B(E2) ↑e^2b^2
118	62	194	1.72	1.7470
120	64	183	1.81	1.8314
122	66	196	1.67	1.6912
124	68	229	1.41	1.4319
126	70	256	1.25	1.2673
128	72	284	1.11	1.1304
130	74	357	0.88	0.8900
132	76	464	0.67	0.6778
134	78	604	0.51	0.5155
136	80	818	0.37	0.3769
138	82	1435	0.210	0.2128
140	84	602	0.50	0.5023
142	86	359	0.82	0.8344
144	88	199	1.47	1.4914
146	90	181	1.60	1.6247
148	92	141	2.03	2.0668

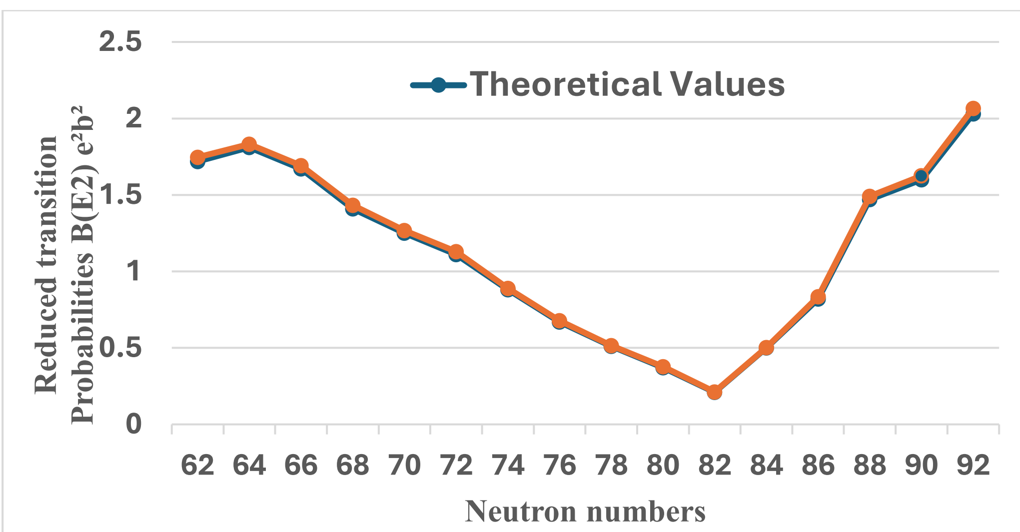

FIGURE 1. Reduced transition probabilities B(E2) for even-even $_{56}$Ba isotopes

TABLE 2. Atomic Number A, Neutron Numbers N, Average Radius Nuclear (R_o^2) & Deformation Parameter (β_2) for $_{56}$ Ba Isotopes.

A	N	Theoretical Values β₂ [15]	Present Work	
			R_o^2	β_2
118	62	-	0.3464	0.2854
120	64	-	0.3503	0.2889
122	66	0.35418	0.3542	0.2746
124	68	0.3027	0.3581	0.2500
126	70	0.2737	0.3619	0.2327
128	72	0.2496	0.3657	0.2174
130	74	0.218315	0.3695	0.1910
132	76	0.1866	0.3733	0.1650
134	78	0.16099	0.3771	0.1424
136	80	0.125812	0.3808	0.1206
138	82	0.093318	0.3845	0.0897
140	84	0.12628	0.3883	0.1365
142	86	0.159542	0.3919	0.1743
144	88	0.1946	0.3956	0.2309
146	90	0.218039	0.3993	0.2388
148	92	-	0.4029	0.2669

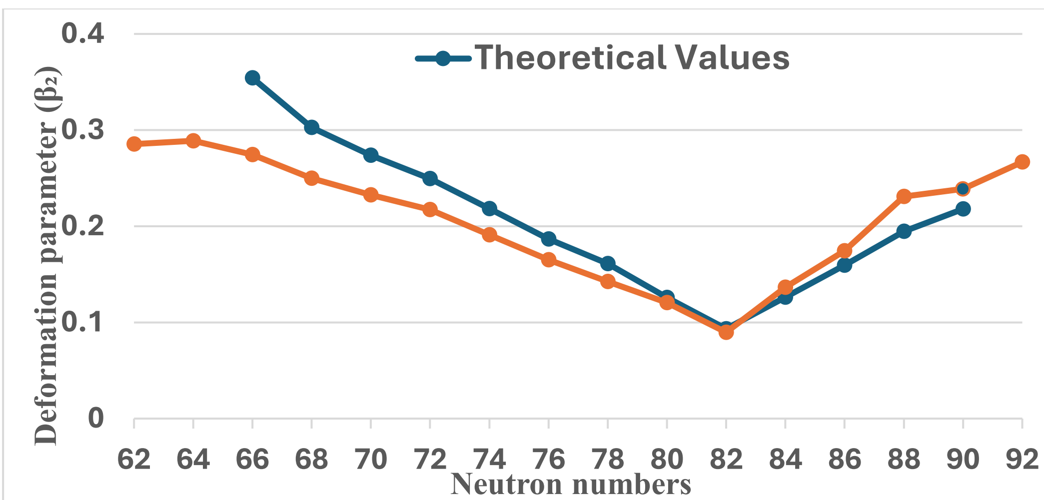

FIGURE 2. Deformation parameter (β_2) for even-even $_{56}$Ba isotopes.

This study focused on studying the effect of neutron numbers on the B(E2)↑ as well as the shape of the nuclide. To determine how the shape of the nuclide is affected by the changing neutron numbers, B(E2)↑ and (β_2) which studied for $_{56}$Ba isotopes, that have neutron numbers (62-92).

From Figure 1, notice that as a result of increasing the number of neutrons, it well decreased in the B(E2)↑ values until it reaches the lowest value when the number of neutrons reaches the magic number (82) (i.e. the relationship is inverse), but after increasing the number of neutrons far from neutron magic number, the relationship becomes directly proportional between the number of neutrons and the B(E2)↑, so we notice a clear increase in the probability of transition with the increase in the neutron numbers.

In Figure 2, the relationship between the calculated values of (β_2) and the number of neutrons was studied by plotting this parameter as a function of the neutron numbers. Here too, there was an inverse relationship between the

(β_2) and the number of neutrons until reaching the magic number of neutrons, where the isotope (^{138}Ba) has the least distortion, which means obtaining a semi-spherical shape.

For isotopes with N > 82 (^{140}Ba to ^{148}Ba), the B(E2)↑ and (β_2) values begin to increase again. This indicates that as neutrons are added beyond the closed shell, they begin to occupy a new, higher-energy shell. These valence neutrons exert a polarizing effect on the spherical core, inducing quadrupole deformation and re-establishing collective rotational behavior. The nucleus moves away from sphericity and becomes progressively more deformed.

On the other hand, nuclei having a deformed charge distribution (deformed shape) when numbers of neutrons move away from a magic number. Nucleus with a magic number of neutrons (full levels) result in a circular symmetry of their charge distribution and circular form of nucleus.

The very low values of B(E2)↑ and (β_2) for ^{138}Ba are a clear and direct signature of this transition to a near-spherical shape. Also, the gamma-ray energy transition for the nucleus (E_{γ_0}) is very high (1435 keV), and the collective motion is minimized, which is characteristic of magic nuclei.

Table-2 explain computed (present work) and theoretical values (earlier work) of (β_2) [15], these values show a clear difference between them, the reason of this variation resulting from adoption Global Best Fit equation for our compute of the B(E2)↑ which is used in computing (β_2), while the reference data marks approved values [15] of B(E2)↑.

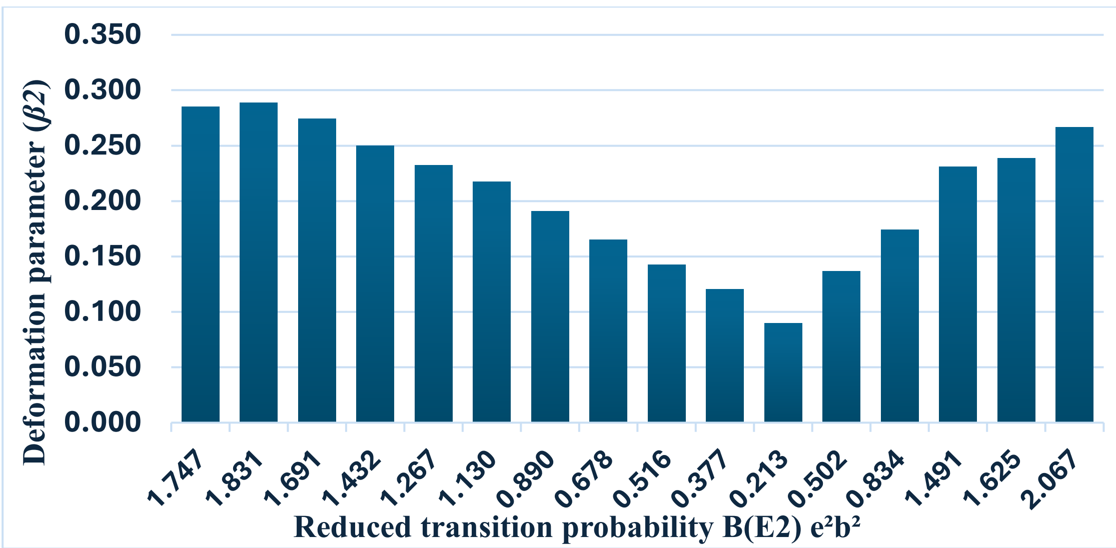

FIGURE 3. Bar chart of the deformation parameter (β_2) with reduced transition probabilities B(E2)↑ for even-even $_{56}Ba$ isotopes.

Figure 3 shows that the lower value of (β_2) (0.0897) is obtained at the minimum value of B(E2)↑(0.2128) for the (^{138}Ba) nucleus, that has a magic neutron number (82). As well as, we can observe the effect of (β_2) by B(E2).

CONCLUSION

This study successfully demonstrated the profound influence of neutron numbers on the collective properties of even-even $_{56}Ba$ isotopes. By applying the Global Best Fit method, we calculated B(E2)↑ and (β_2) for $_{56}Ba$ isotopes from which have neutron numbers (N=62-92).

The primary conclusion is that the nuclear structure undergoes a significant transformation at the neutron magic number N=82. The minima in both B(E2)↑ and (β_2) at ^{138}Ba provide strong evidence for a transition from deformed, collective nuclei to a more spherical, non-collective configuration at the closed shell. The subsequent increase in these values for N > 82 confirms the re-emergence of deformation as neutrons are added to the next shell. This systematic analysis reinforces the fundamental concepts of nuclear shell theory and collective motion, highlighting the delicate balance of forces that dictate the shape of the atomic nucleus.

REFERENCES

1. A. Aguilar, High-Spin Nuclear Structure of ^{168,170}Ta and Triaxial Strongly Deformed Structure in ^{160}Yb, 2008.
2. P. Doornenbal et al., In-beam gamma-ray spectroscopy of 34,36,38 Mg: Merging the N = 20 and N = 28 shell quenching, Physical Review Letters 111,21,212502, (2013).
3. L. John, Nuclear physics principles and application (Wiley and Sons, 2001), pp. 45–61.
4. L.J. Basdevant, R. James, and S. Michel, Fundamentals in Nuclear Physics, (Springer Science Business Media, Palaiseau, France, 2005).
5. A. H. Ali and M. T. Idrees, Study of Deformation Parameters (β_2, δ) For 18,20,22,24,26,28 Ne isotopes in sdpf shell, Karbala International Journal of Modern Science: Vol. 6: Iss. 1, Article 11 (2025). doi.org/10.33640/2405-609X.1376.
6. J.P. Delaroche et al., Structure of Even-Even nuclei Using a Mapped Collective Hamiltonian and the D1S Gogny interaction, Phys. Rev. C 81, 014303 (2010).
7. W. Pfeifer, An Introduction to the Interacting Boson Model of the Atomic Nucleus (vdf Hochstetler ag an der ETH Zurich, 1998).
8. S. Akkoyun, T. Bayram, and S.O. Kara, A Study on Estimation of Electric Quadrupole Transition Probability in Nuclei, Journal of Nuclear Sciences 2(1),7-10, (2015).
9. A. Al-Sayed and A. Y. Abul-Magd, Level Statistics of Deformed Even-Even Nuclei, Phys. Rev. C 74, 037301 (2006).
10. A. A. Ridha, Deformation Parameters and Nuclear Radius of Zirconium (Zr) Isotopes Using the Deformed Shell Model, Journal of Wasit for Science and Medicine 2(1), 115–125 (2009).
11. Salame and P. Mialhe, “N-channel power MOSFET for fast neutron detection,” Microelectron. Int. 19, 19–22 (2002).
12. N. T. Jarallah and H. J. Hassan, Determination of the shape for (54Xe and 82pb) nuclei from deformation parameters (β_2, δ), Iraqi Journal of Science 57(3B), 2014–2024 (2016).
13. M. S. Nayyef, Study of nuclear properties for even-even nuclei (Transition probabilities B(E2)↑, deformation parameters β_2 and quadrupole moments Q_o, International Journal of Applied Research,11(8):379-384(2025).
14. M. M. Hamarashid, Determination Multipole Mixing Ratios and Transition Strengths of Gamma Rays from Level Studies of ^{93}Mo (p,nγ) Reaction, Journal of Physical Science and Application 2(7), 253–257 (2012).
15. S. Raman, C.W. Nestor, and JR. Tikkanen, Transition Probability from the Ground to the First-Excited 2^+ State of Even–Even Nuclides, Atomic Data and Nuclear Data Tables 78, 1–128 (2001).
16. N. T. Jarallah and M. S. Nayyef, Study the Relationship between Beta Decay Stability of Nuclide and its Shape for Some even-even Isobars, J. Phys. Conf. Ser. 1879(3), 032103 (2021).
17. A. Boboshin, B. Ishkhanov, S. Komarov, V. Orlin, N. Peskov, and V. Varlamov, Investigation of Quadrupole Deformation of Nucleus and its Surface Dynamic Vibrations, International Conference on Nuclear Data for Science and Technology, DOI: 10.1051/ndata:07103, 65–68 (2007).
18. R.B. Firstone and V.S. Shirley, Table of Isotopes, Eight Edition, (John Wiley and Sons, 1999).