**Linear Natural Vibrations of a Viscoelastic Toroidal Shell**

F.B. Jalolov1, a), B.S. Rahmonov2, b), S.J. Sobirov1, c), N.A. Narzullayev 1, d) and M.A. Ruzieva1, e)

1*Bukhara State Technical University, 15 Murtazaev Street, Bukhara 200100, Uzbekistan*2*Urgench State University, 14 Kh. Alimdjan Street, Urgench 220100, Uzbekistan*

*a) Corresponding author:* [*farruhjalolov2@gmail.com*](mailto:farruhjalolov2@gmail.com) *b)*[*bah-bahodir@yandex.ru*](mailto:bah-bahodir@yandex.ru) *c)* [*sobirovsobir19771@gmail.com*](file:///D:\Конф.%20Транспорт%20Унив\23.07.2025\sobirovsobir19771@gmail.com) *d)* [*muxammadnarzulloyev15@gmail.com*](mailto:muxammadnarzulloyev15@gmail.com) *e)* [*ruziyevama825@gmail.com*](mailto:ruziyevama825@gmail.com)

**Abstract.** In practice, pipelines are used for liquid transportation. In this paper, linear vibrations of a viscoelastic toroidal shell with different physical and mechanical properties are considered. Numerical results are obtained on the basis of Muller's method. The torus is a closed surface and therefore the stiffness of such a shell is much higher than that of a cylindrical shell. The greater stiffness of a torus compared to a cylinder will affect not only the absolute values of frequencies, but also the character of their distribution. When finding the frequencies of natural vibrations we make use of the Ritz method. In case the lowest natural frequency of a cylindrical shell is determined by the minimum number of waves in the longitudinal direction, this is not the case of a torus. It is found that the main role in the bending vibrations of thin-walled shells is played by the displacement, which for a torus is accounted for by both the stretching and bending energies, in contrast to a cylinder, where the stretching energy is determined only by the longitudinal displacement.

**Keywords:** vibrations, viscoelastic shell, Mueller's method, natural vibrations, number of waves

**INTRODUCTION**

The practicality of liquid transportation pipe relies on rigid masses, which are widely used in various fields. As an example, aerospace industry: transport pipes of the fuel, power industry: transport pipes of oil and gas, nuclear industry: heat exchangers. Unwanted vibration frequently arose at the working environment and flow of fluids through the pipe. Thus, it has always been an attention of the researchers to understand in detail about the vibration modes of different pipes used to transport fluids [1, 2, 3, 4]. Majority of works on vibration characterization have been devoted to freely supported or cantilevered pipes. Fluid transporting pipes with diverse boundary constraints will also have differences in vibrations.

Dynamic stability of the pipes handling fluid systems with diverse boundary limitations coupled with the conditions of divergence and flutter of pipes were revealed in [5]. Dependent upon the constraint analysis [6, 7, 8], nonlinear equations of motion of pipes in fixed-free and fixed-fixed ends were determined. Cantilevered pipes conveying fluid lots of dynamic phenomena have simple boundary conditions, owing to the free boundary at one end.

Subharmonic and compound resonances of a pipe that moves a pulsating fluid were investigated in [9, 10, 11]. Another problem of dynamic calculation and taking into account real edge fixations of separate pipeline sections [12, 13] and the choice of approximating functions satisfying the given boundary conditions are considered in [14, 15].

**STATEMENT OF THE PROBLEM AND THE METHODS OF SOLVING THE PROBLEM**

The shell is formed by rotation of a ring (see Fig. 1) of radius r with respect to the axis О. The distance of the center of the ring from the axis is denoted by . Since there are no theoretical or experimental data in the literature on the frequency spectrum of natural vibrations, in practical calculations the torus is sometimes replaced by a cylindrical shell whose length is equal to the average perimeter of the torus, and the radius r- is equal to the smaller radius of the torus. The lowest frequency is related not only to geometrical characteristics, but also to a certain number of waves in the and directions.

The radius of a section arbitrarily taken will be:

(1)

The position of a point on the torus surface is defined by the system of orthogonal curvilinear coordinates and . The geometrical parameters of the torus in the notations [2] will be:

(2)

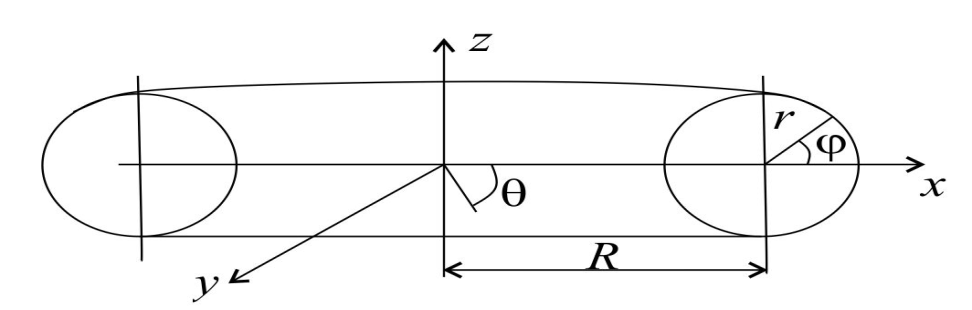
To obtain the deformation parameters, let us denote: – is the displacement along the normal to the medial surface; - is the displacement of the tangent coordinate ; - is the displacement of the tangent coordinate . Based on the general formulas of shell theory, the tensile strain parameters will be:

(3)

The bending parameters will be:

(4)

The formulas (4) are valid not only for a closed torus, but also for any toroidal surface.



**FIGURE 1.** Calculation scheme

**SELECTION OF APPROXIMATING FUNCTIONS**

Ritz method is applied to obtain natural vibration frequencies of a toroidal shell, and in the analysis of cylindrical shells and conical shells. The success of this method is largely determined by the choice of approximating functions. It is reasonable to define the displacement in the form of a double series of functions, namely

(5)

Let define the vibrations of the elementary ring of radius ; - vibrations of the elementary ring of radius - integer numbers.

By imposing on the displacements additional conditions , we obtain

where

We determine the deformation parameters for the selected functions:

(6)

**COMPILATION OF THE CHARACTERISTIC EQUATION**

Having determined the deformation parameters and made expressions of potential P and kinetic T energies, we find, differentiating P and T by , the coefficients of the characteristic equations. The system of functions depending on , is orthogonal. Therefore, the following condition is satisfied

(7)

where

. (8)

The coefficients after the simplest algebraic transformations can be written as follows:

(9)

The last integral is taken in series and for small and large (this term can be neglected). Integration of the coefficients presents considerable computational difficulties. The integrals are not defined in closed form and must be calculated using series. Since is small, , only the terms can be retained in the coefficients of :

(10)

(11)

However, in the diagonal terms, the value of cannot be neglected compared to the value of , since the order of smallness is the same for a number of values of n and In the side terms, only the values of can be retained. In formulas (12) and (13), the parameters have the following expressions

(12)

**RESULTS AND ANALYSIS**

For ease of calculation, we present the values of, which are given in Table 1.

**TABLE 1.** Values of at differentand

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  | 5 | 6 | 8 | 9 | 10 |
|  |  | 3,17  -3,20 | 3,11  -3,06 | 3,06  -2,96 | 3,05  -2,93 | 3,04  -2,93 |
|  |  | 1,60  -1,18 | 1,59  -1,13 | 1,58  -1,09 | 1,58  -1,08 | 1,58  -1,08 |

The following approximate formula can be used to estimate the frequency of natural oscillations

(13)

or expanded

. (14)

For small, the frequency of natural oscillations is practically independent of m number of waves along the radius of the larger circle. For small , т.е., if the frequency calculation follows the formula for an infinitely long cylindrical shell. Based on this study, we conclude that as decreases, the frequencies of the cylindrical and toroidal shells will converge. For small n, the difference in frequencies will be significant due to the difference in coefficients.

**CONCLUSIONS**

It is found that the main role in bending vibrations of thin-walled shells is played by the displacement , which for a torus is accounted for by both tensile and bending energies, in contrast to a cylinder, where the tensile energy is determined only by the longitudinal displacement . The formulas for a cylindrical shell can be used only for .

**FUTURE SCOPE**

The study mainly deals with the linear analysis of natural vibration of the toroidal shells which are constituted with the viscoelastic material, where they take into account idealized boundary and loading conditions. Nevertheless, there are a few prospective avenues of developing this research: Nonlinear Dynamic Behavior: Geometric, as well as material nonlinearities, would then be included and would permit the real time behavior under large deformations or complex excitations to be predicted more accurately. Temperature-Dependent Material Properties: As viscoelastic materials are susceptible to responding to changes in temperature, this can be incorporated in future investigations of how the thermal fields effects the vibration properties. Multilayer and Functionally Graded Structures: The aim of exploring multilayered Toroidal shells and functionally graded viscoelastic materials is to enhance the performance of vibrations and damping characteristics of certain engineering purposes. Experimental Validation: The development of experimental inferiors to validate theoretical and numerical models will help to enhance the confidence of vibration properties of the structure and allow the optimization of material dissimilarities of viscoelastic materials through experimental validation. Optimization Studies: The studies about the optimization of the design in terms of the geometry of the shell, the distribution of the thickness, and the material composition to optimize the issues of resonance or damping in problematic frequency bands can also be the future work. Fluid-Structure Interaction (FSI): When there are applications where a fluid-filled toroidal shell or submerged toroidal shells are involved it would be possible to include FSI into the vibration analysis and give more realistic results to structures in marine, biomedical or aerospace applications. Developed Computational Methods: Applying machine learning or reduced-order based methods can substantially lower the computational cost and can be built to perform assessments of the complex shell geometry quickly. These directions in the future can broaden the comprehension of toroidal viscoelastic shell models to the contemporary engineering systems such as structures sensitive to vibration, damping mechanisms as well as precision parts within the aerospace and biomedical engineering arena.

**REFERENCES**

1. V. I. Pozhuev, The action of a moving load on a cylindrical shell in an elastic medium, Construction Mechanics of Computed Structures, No. 1, 44–48 (1978). doi:10.1007/BF00884609.
2. V. P. Maiboroda, I. E. Troyanovskii, I. I. Safarov, G. M. Vazagashvili, and I. V. Katalymova, Wave attenuation in an elastic medium, Journal of Soviet Mathematics **60**, 1379–1382 (1992). doi:10.1007/BF01679642.
3. S. A. Bochkarev, S. V. Lekomtsev, and V. P. Matveenko, Natural vibrations and stability of elliptical cylindrical shells containing fluid, International Journal of Structural Stability and Dynamics **16**, 1550076 (2016).
4. M. Amabili, Free vibration of partially filled, horizontal cylindrical shells, Journal of Sound and Vibration **191**, 757–780 (1996). doi:10.1006/jsvi.1996.0154.
5. V. P. Maiboroda, I. I. Safarov, and I. E. Troyanovskii, Free and forced oscillations of a system of rigid bodies on inhomogeneous viscoelastic snubbers, Soviet Machine Science (English translation) **3**, 25–31 (1983).
6. I. I. Safarov and M. Kh. Teshaev, Unsteady motions of spherical shells in a viscoelastic medium, Bulletin of Tomsk State University. Mathematics and Mechanics (June 2023), online first.
7. I. I. Safarov and M. Kh. Teshaev, Nonlinear oscillations in the dynamics of multilayer viscoelastic shells, News of Higher Educational Institutions. Applied Nonlinear Dynamics **31**, 63–73 (2023).
8. I. I. Safarov, M. Kh. Teshaev, A. M. Marasulov, and B. Z. Nuriddinov, Propagation of own non-axisymmetric waves in viscoelastic three-layer cylindrical shells, Engineering Journal **25**, 97–107 (2021). doi:10.4186/ej.2021.25.7.97.
9. M. Kh. Teshaev, I. I. Safarov, N. U. Kuldashov, M. R. Ishmamatov, and T. R. Ruziev, On the distribution of free waves on the surface of a viscoelastic cylindrical cavity, Journal of Vibration Engineering and Technologies **8**, 579–585 (2020).
10. M. Kh. Teshaev, I. I. Safarov, and M. Mirsaidov, Oscillations of multilayer viscoelastic composite toroidal pipes, Journal of the Serbian Society for Computational Mechanics **13**, 105–116 (2020). doi:10.24874/jsscm.2019.13.02.02.08.